Threshold Computation in the Head

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New Trends in PQC Workshop

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Threshold Computation in the Head

Joint work with Thibauld Feneuil

https://ia.cr/2022/1407

Original TCitH framework (Asiacrypt'23)

https://ia.cr/2023/1573

Improved TCitH framework (preprint)

Roadmap

- MPC-in-the-Head paradigm
- TC-in-the-Head framework (and application to PQ signatures)
  - 🌲 TCitH with Merkle trees
  - 🌲 TCitH with GGM trees
  - ✖ TCitH using multiplication homomorphism
  - 📦 TCitH using packed secret sharing
- Application: post-quantum ring signatures
- Relation to other proof systems

MPC-in-the-Head paradigm

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One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

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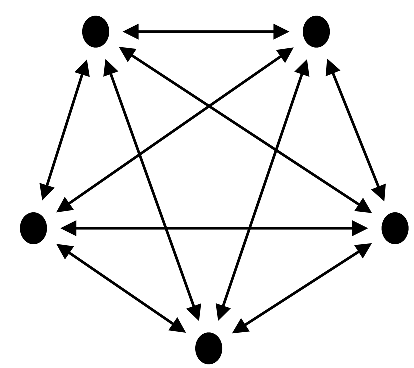
Multiparty computation (MPC)



Input sharing $[\![x]\!]$

Joint evaluation of:

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

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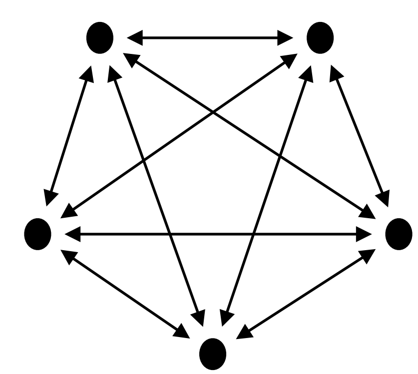

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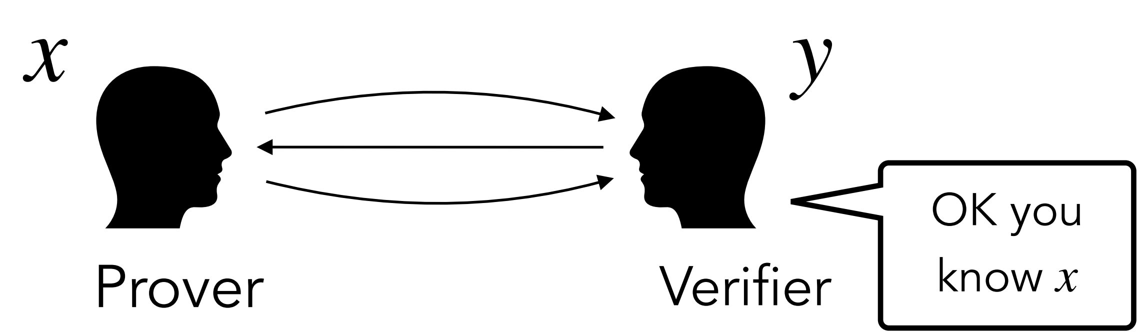
Joint evaluation of:

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

Zero-knowledge proof



MPC-in-the-Head paradigm

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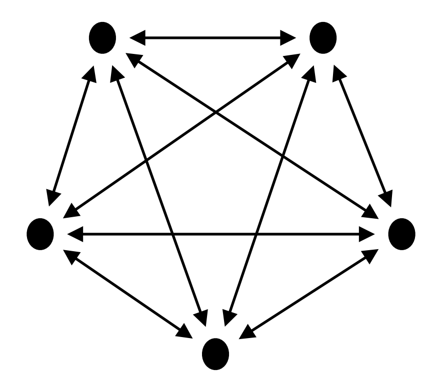

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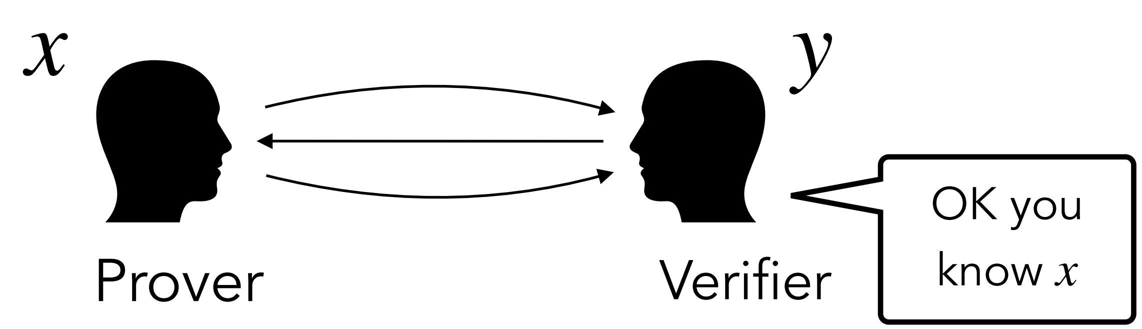
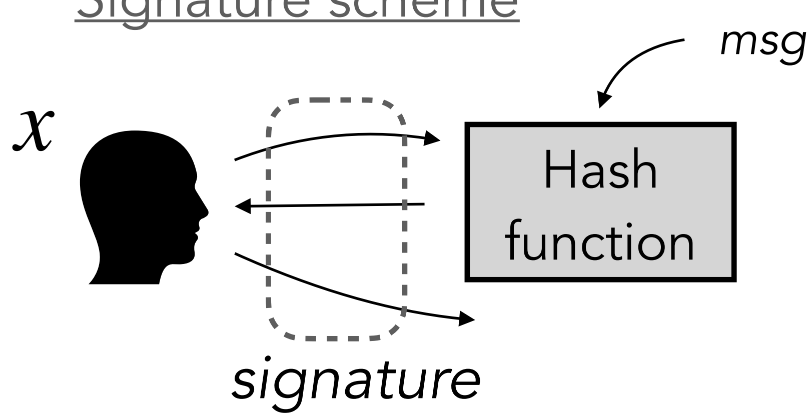
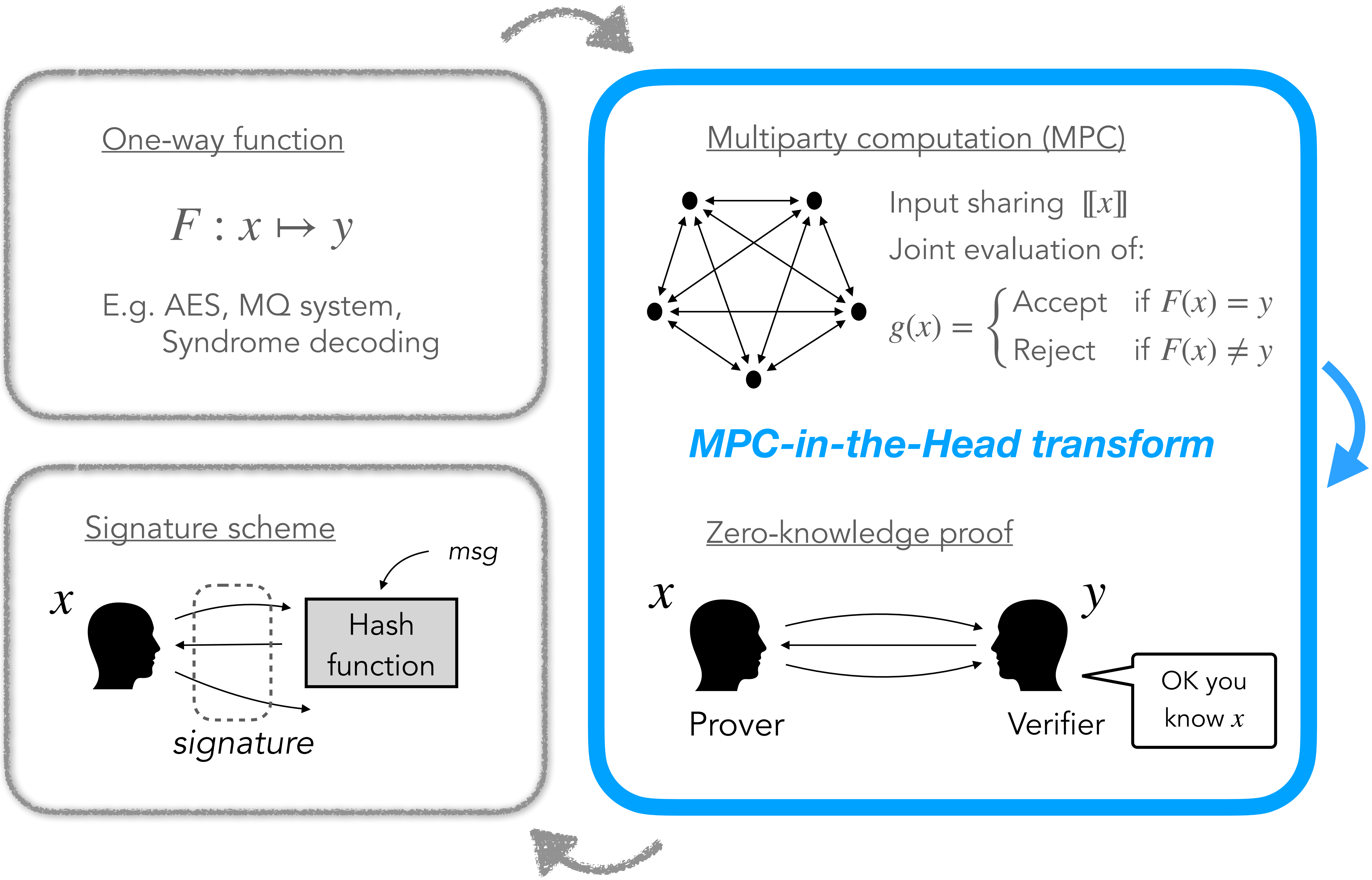


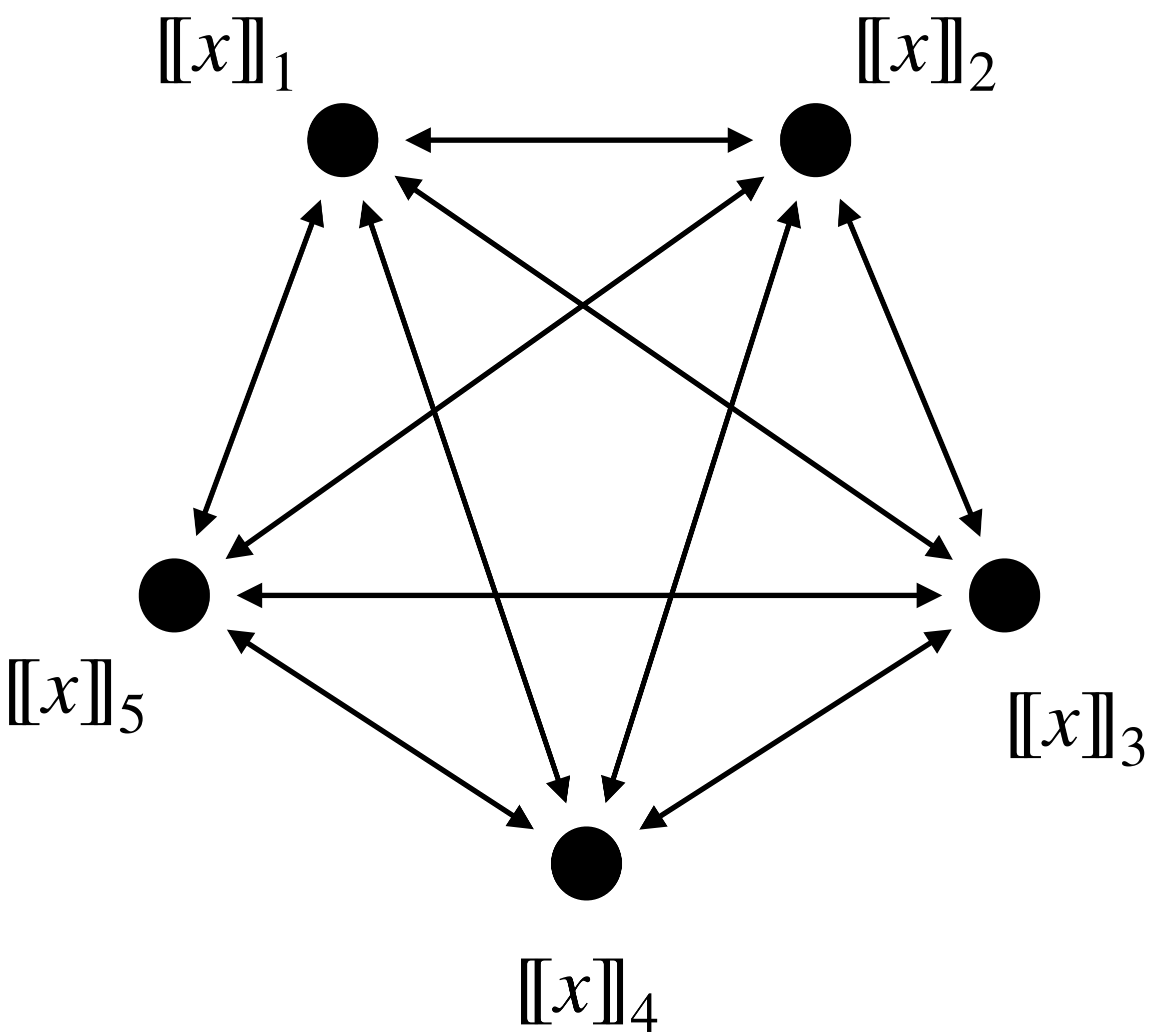
Signature scheme



MPC-in-the-Head paradigm



MPC model



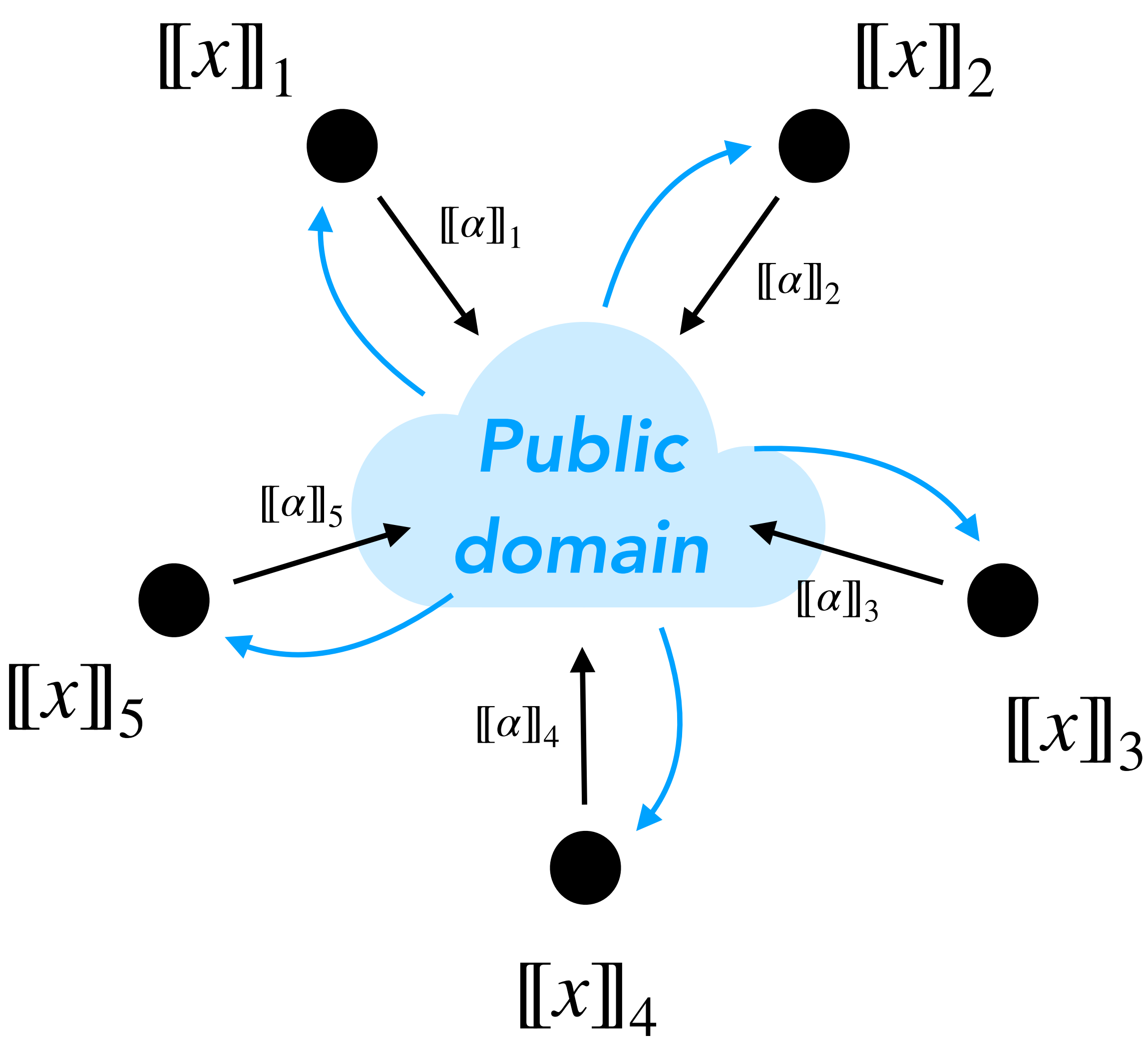
- Jointly compute

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

- ℓ-private
- Semi-honest model

$[\![x]\!]$ is a linear secret sharing of x

MPC model



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- ℓ-private
- Semi-honest model
- *Broadcast model*

MPCitH transform

Prover

Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \dots, [\![x]\!]_N)$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\dots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

Prover

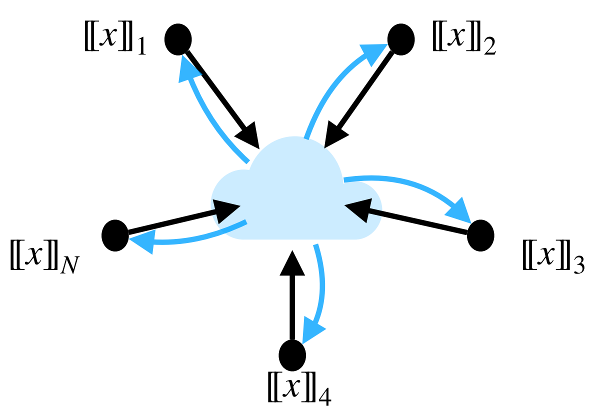
Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

② Run MPC in their head



$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\cdots$

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send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

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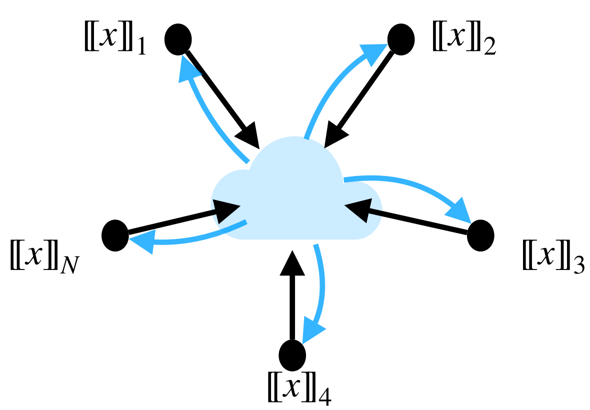

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send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

I

③ Choose a random set of parties

$I \subseteq \{1,\ldots,N\}$, s.t. $|I| = \ell$.

Prover

Verifier

MPCitH transform

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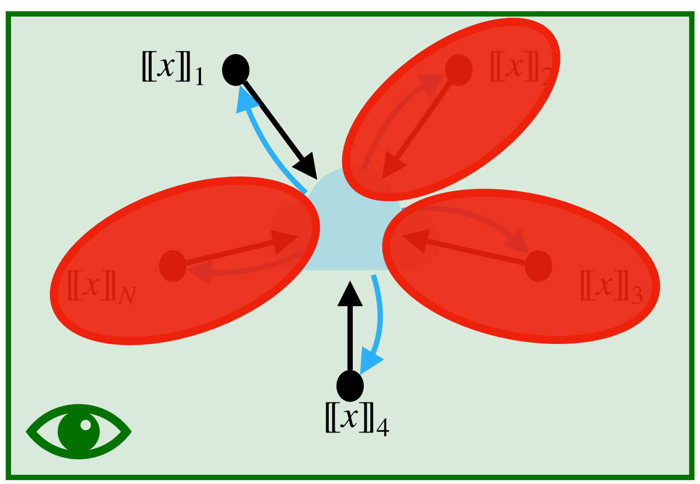

④ Open parties in I

$\text{Com}^{\rho_1}([\![x]\!]_1)$

$\cdots$

$\text{Com}^{\rho_N}([\![x]\!]_N)$

→

send broadcast

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→

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←

$([\![x]\!]_i, \rho_i)_{i \in I}$

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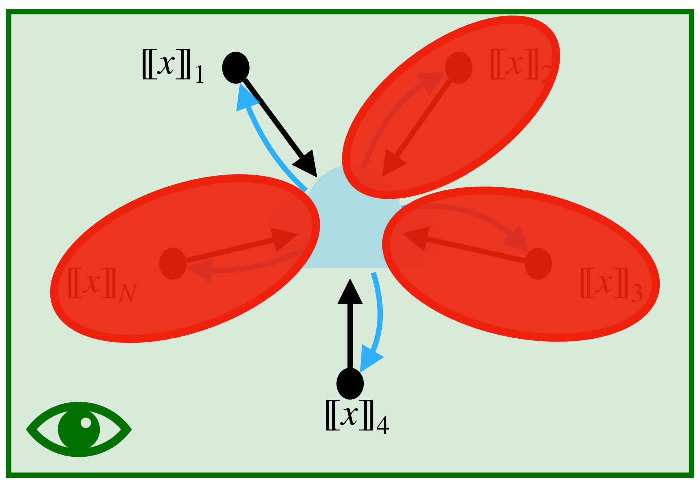

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send broadcast
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⑤ Check $\forall i \in I$
- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $g(y, \alpha) =$ Accept

Prover

Verifier

MPCitH transform: with additive sharing

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$[\![x]\!] = ([\![x]\!]_1, \dots, [\![x]\!]_N)$

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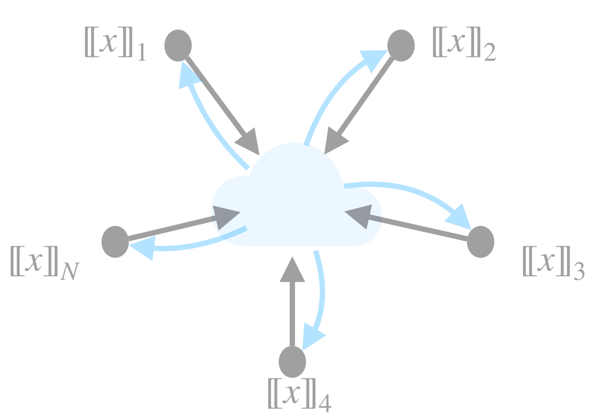

④ Open parties in I

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$
$\dots$
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

send broadcast

Additive sharing:
$x = [\![x]\!]_1 + \cdots + [\![x]\!]_N$

$([\![x]\!]_i, \rho_i)_{i \in I}$

Choose a random set of parties $I \subseteq \{1,\dots,N\}$, s.t. $|I| = \ell$.

⑤ Check $\forall i \in I$
- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

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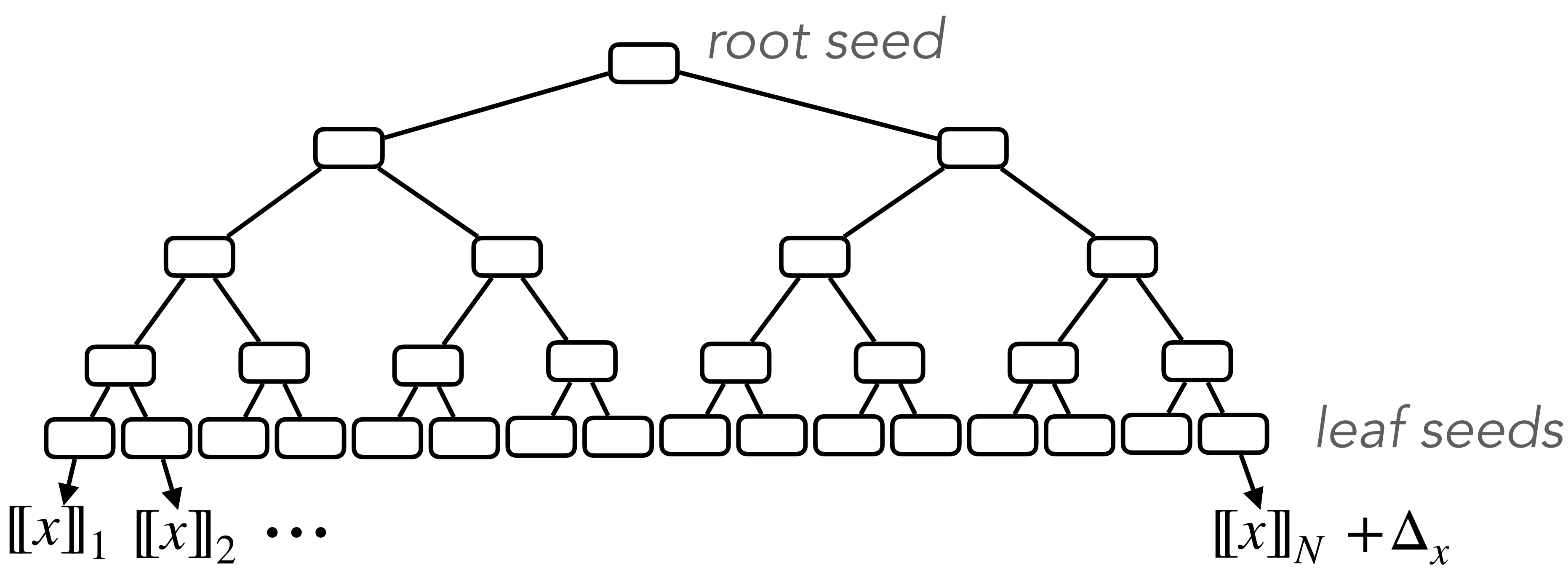
② Run MPC in their head

Generated using a GGM seed tree [KKW18]:



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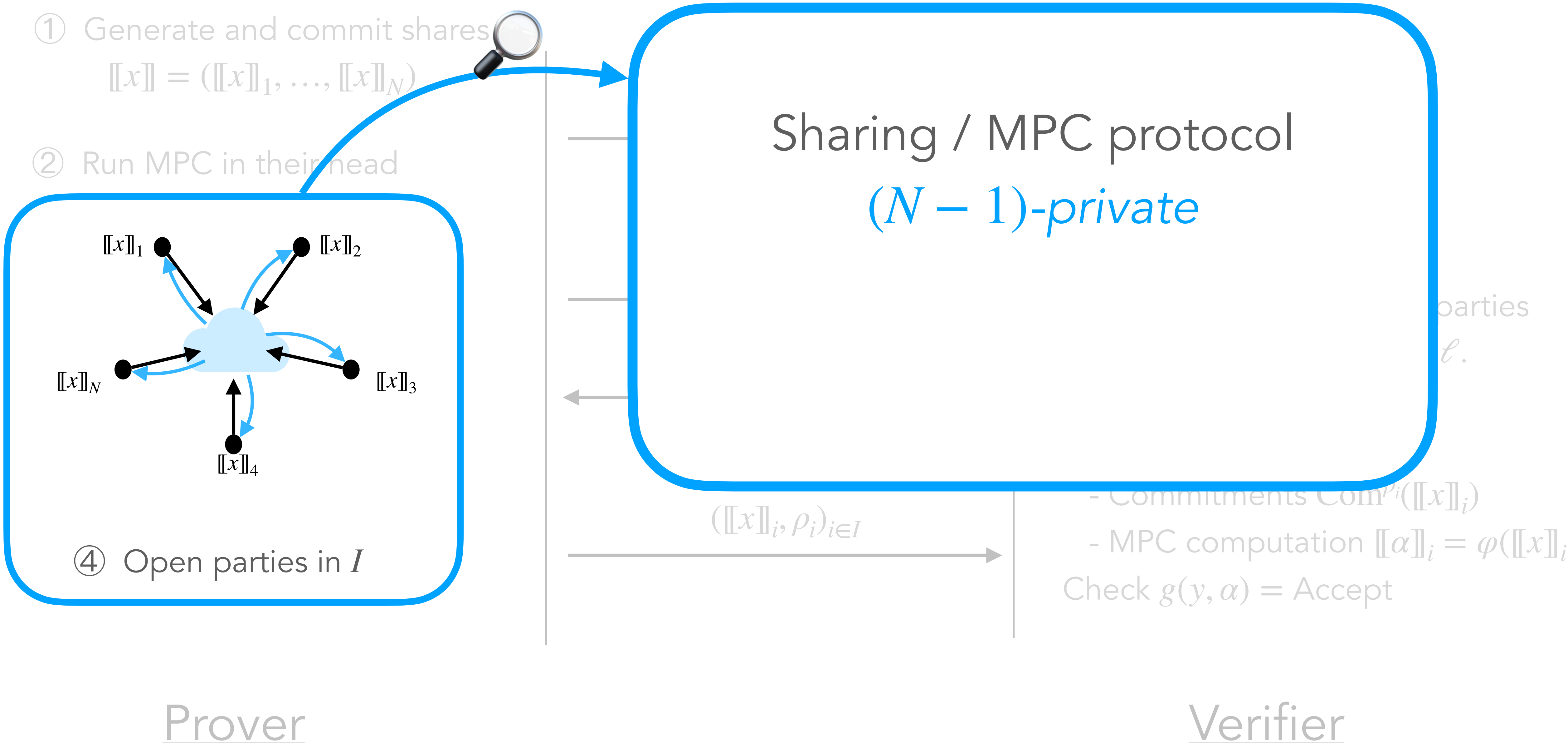

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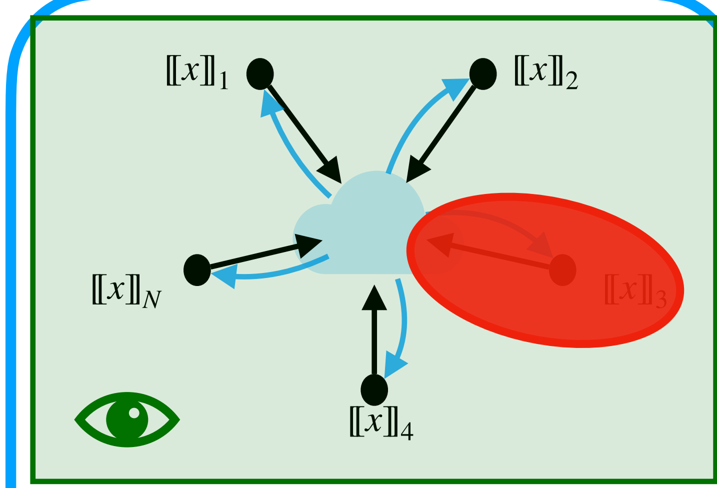

④ Open parties in I

Sharing / MPC protocol

$(N-1)$-*private*

parties

ℓ.

$([\![x]\!]_i, \rho_i)_{i \in I}$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
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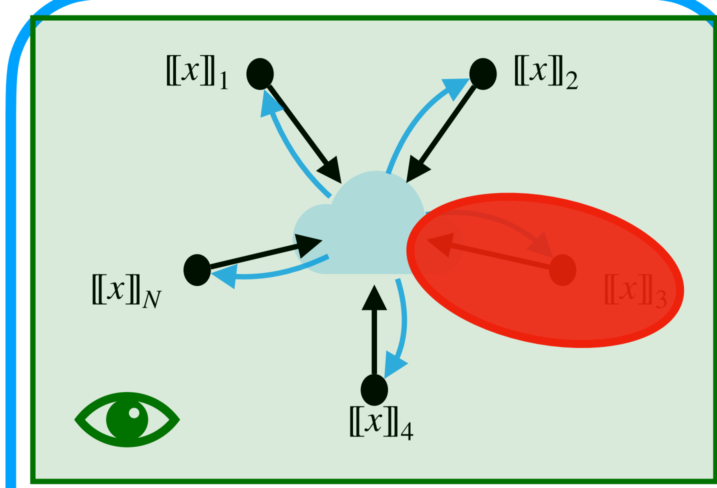

④ Open parties in I

Sharing / MPC protocol

$(N-1)$-*private*

$\Rightarrow$ soundness error $= \dfrac{1}{N}$

parties

ℓ.

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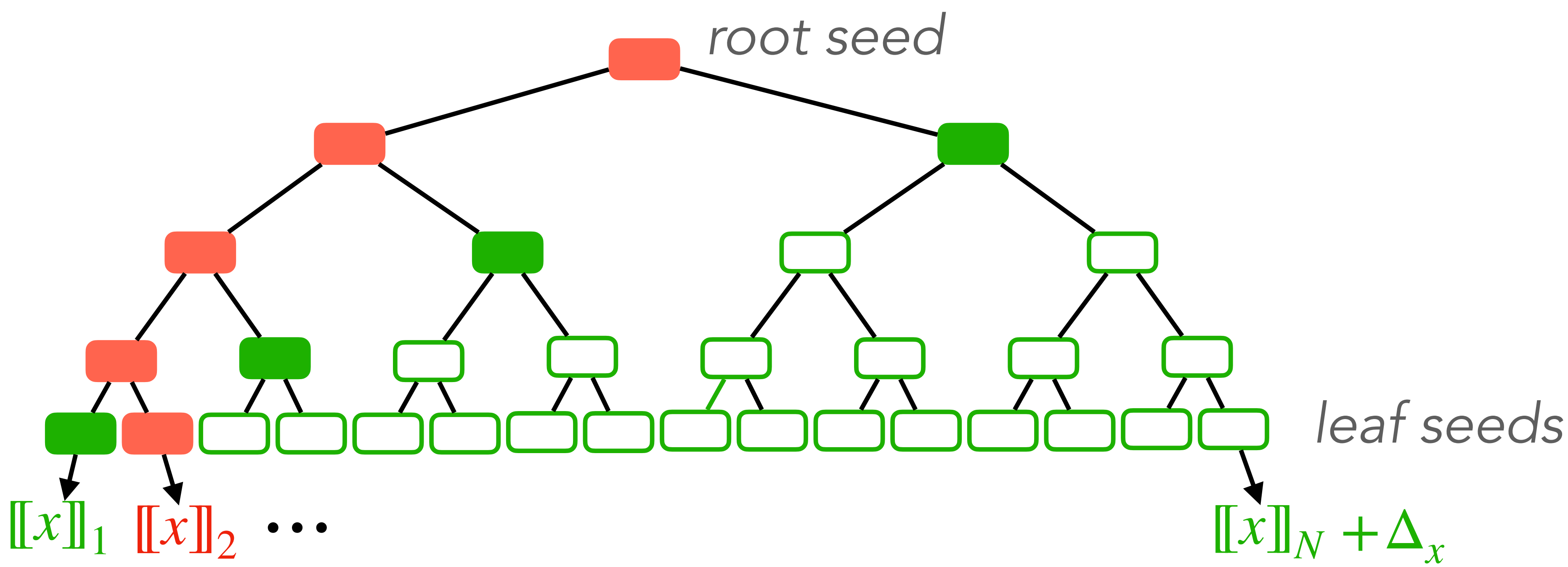
Only $\log_2 N$ seeds to be revealed:



TC-in-the-Head framework (with Merkle trees)

Threshold Computation in the Head

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$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$
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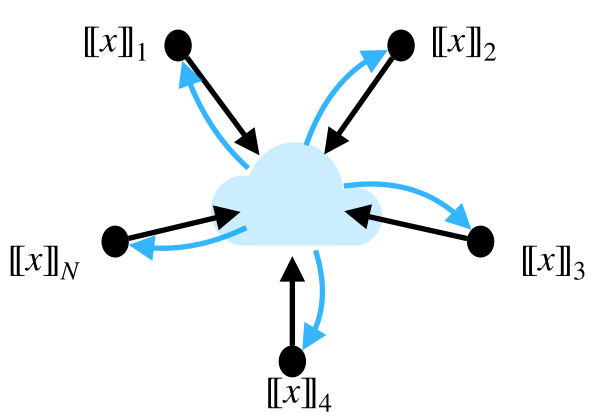
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$ →

② Run MPC in their head



send broadcast
$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$ →

③ Choose a random set of parties
$I \subseteq \{1, \ldots, N\}$, s.t. $|I| = \ell$.

← I

④ Open parties in I

$([\![x]\!]_i, \rho_i)_{i \in I}$ →

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Prover | Verifier

Threshold Computation in the Head

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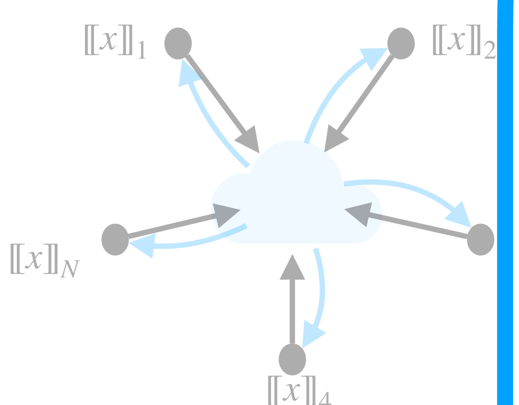
② Run MPC in their hea



④ Open parties in I

Shamir secret sharing:

$$[\![x]\!]_i := P(e_i) \quad \forall i$$

$$\text{for } \; P(X) := x + r_1 \cdot X + \cdots + r_\ell \cdot X^\ell$$

n set of parties
t. $|I| = \ell$.

$\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
ion $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$
ccept

Prover

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Threshold Computation in the Head

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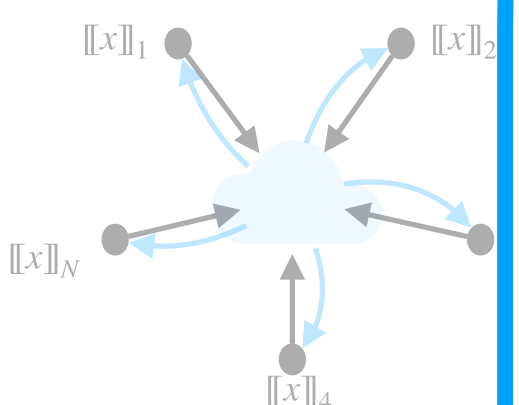

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$\Rightarrow$ ℓ-privacy

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Threshold Computation in the Head

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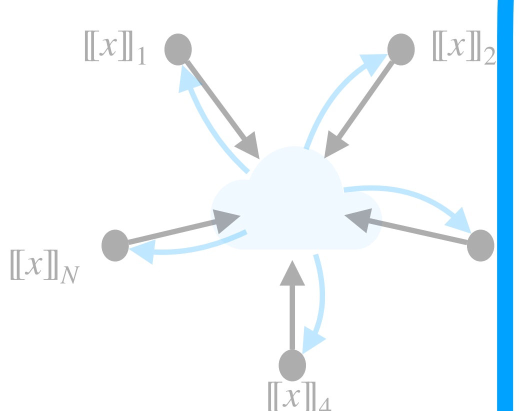

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We use $\ell \ll N$ (e.g. $\ell = 1$)

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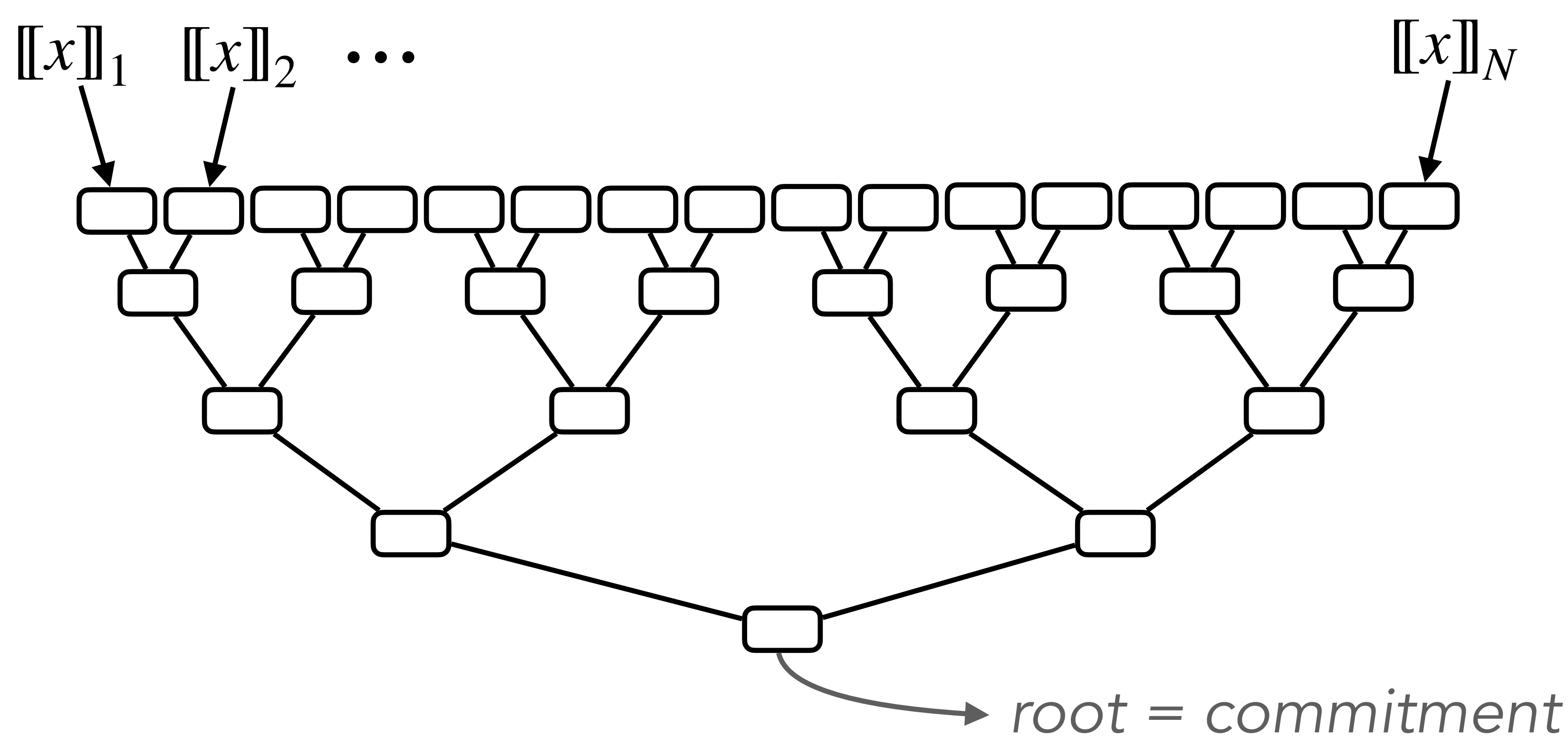
② Run MPC in their head

Committed using a Merkle tree:

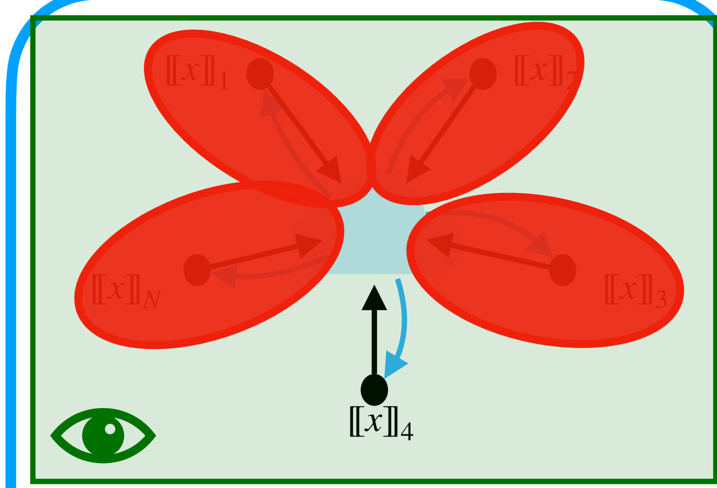


Threshold Computation in the Head

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

② Run MPC in their head



④ Open parties in I

Sharing / MPC protocol *ℓ-private*

parties

ℓ .

$]\!]_i)$

$= \varphi([\![x]\!]_i)$

Prover

Verifier

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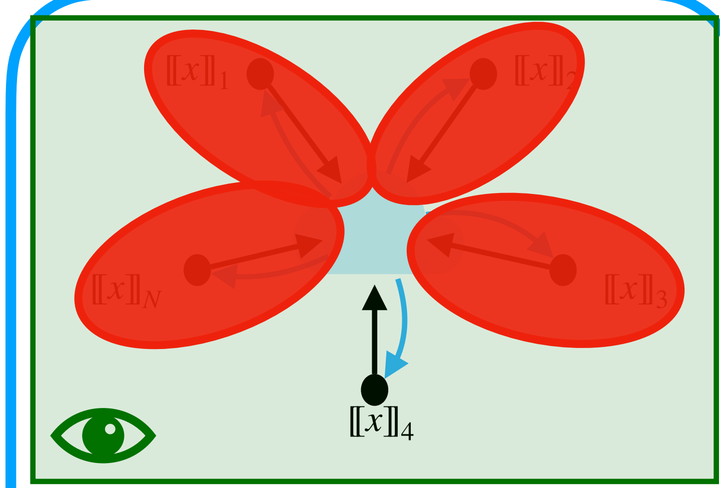

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Sharing / MPC protocol *ℓ-private*

$\Rightarrow$ soundness error = $(N - \ell)/N$ 🤔

Prover

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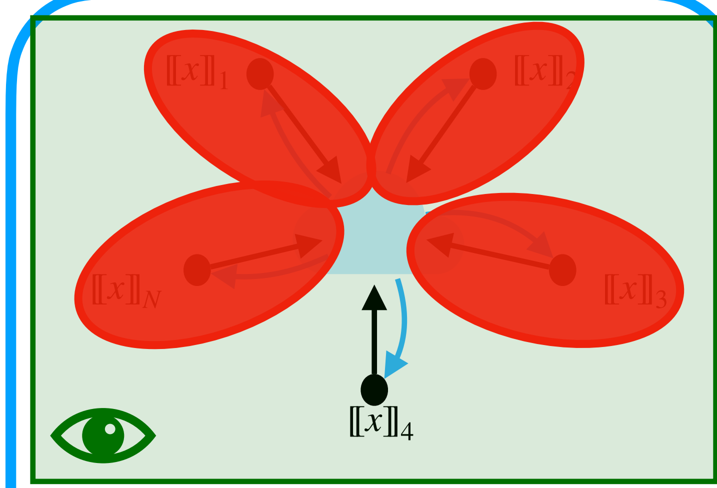

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Sharing / MPC protocol *ℓ-private*

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💡 broadcast messages must be valid Shamir's sharings

Prover

Threshold Computation in the Head

① Generate and commit shares $[\![x]\!] = ([\![x]\!]_1, \dots, [\![x]\!]_N)$

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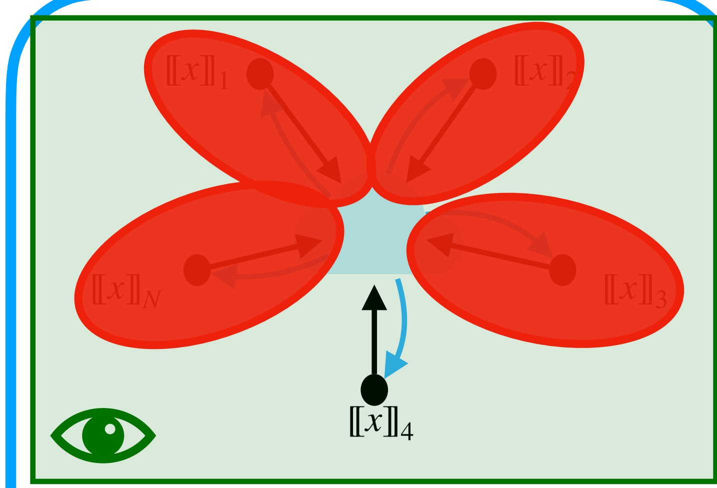


④ Open parties in I

Prover

Sharing / MPC protocol *ℓ-private*

~~⇒ soundness error = $(N-\ell)/N$~~ 🤔

💡 broadcast messages must be valid Shamir's sharings

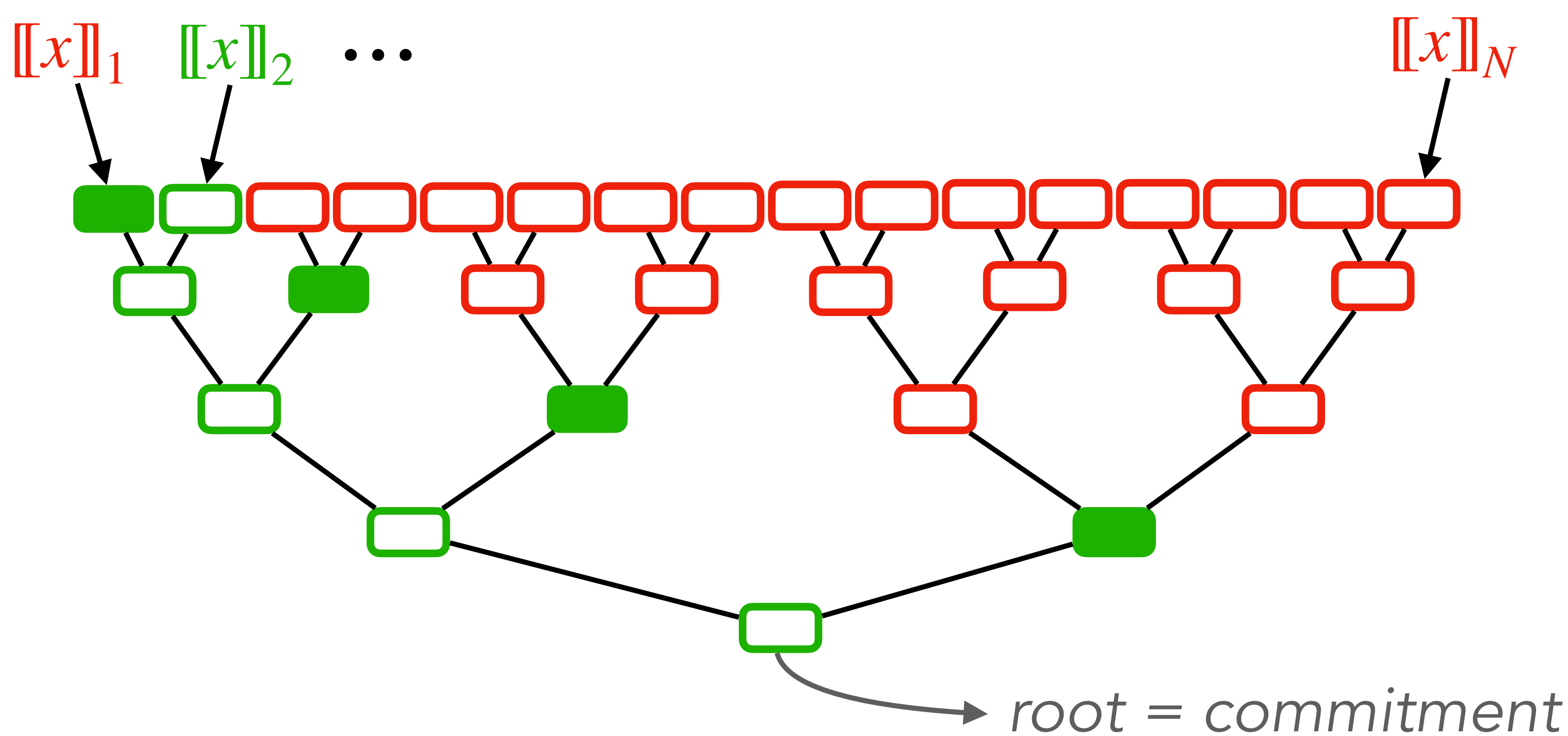
⇒ soundness error = $\dfrac{1}{\binom{N}{\ell}}$ 🤩

Verifier

Threshold Computation in the Head

① Generate and commit shares $\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

Only $\log_2 N$ labels to be revealed:



Soundness

$$p = \text{"false positive probability"}$$
$$= P\left[\text{MPC protocol accepts } [\![x]\!] \text{ while } f(x) \neq y\right]$$

Soundness

p = "false positive probability"

$= P\big[$MPC protocol accepts $[\![x]\!]$ while $f(x) \neq y\big]$

$$\frac{1}{N} + p$$

Soundness error of standard MPCitH

Soundness

p = "false positive probability"

$= P\big[\text{MPC protocol accepts } [\![x]\!] \text{ while } f(x) \neq y\big]$

hope 🙏

$$\frac{1}{N} + p \quad \longrightarrow \quad \frac{1}{\binom{N}{\ell}} + p$$

Soundness error of standard MPCitH

Soundness error of TCitH

Soundness

$$p \;=\; \text{"false positive probability"}$$
$$\;=\; P\big[\text{MPC protocol accepts } [\![x]\!] \text{ while } f(x) \neq y\big]$$

$$\frac{1}{N} + p$$

Soundness error of standard MPCitH

hope 🙏 → ~~$\frac{1}{\binom{N}{\ell}} + p$~~

Soundness error of TCitH

reality 😬 → $$\frac{1}{\binom{N}{\ell}} + p \cdot \frac{\ell(N-\ell)}{\ell+1}$$

Soundness

$$\frac{1}{\binom{N}{\ell}} + p \cdot \frac{\ell(N-\ell)}{\ell+1}$$

Why?

Soundness

$$\frac{1}{\binom{N}{\ell}} + p \cdot \frac{\ell(N-\ell)}{\ell+1}$$

Why?

- Prover can commit invalid sharings
- Let $[\![x]\!]^{(J)}$ = sharing interpolating $\left([\![x]\!]_i\right)_{i \in J}$
- Many different $[\![x]\!]^{(J)} \Rightarrow$ many possible false positives

Soundness

$$\frac{1}{\binom{N}{\ell}} + p \cdot \frac{\ell(N-\ell)}{\ell+1}$$

Why?

- Prover can commit invalid sharings
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- Many different $[\![x]\!]^{(J)} \Rightarrow$ many possible false positives

- "Degree-enforcing commitment scheme"
- Verifier $\rightarrow$ Prover : random $\{\gamma_j\}$
- Prover $\rightarrow$ Verifier : $[\![\xi]\!] = \Sigma_j \, \gamma_i \cdot [\![x_j]\!]$
- Before MPC computation

Soundness

$$\frac{1}{\binom{N}{\ell}} + p \cdot \frac{\ell(N-\ell)}{\ell+1}$$

Why?

- Prover can commit invalid sharings
- Let $[\![x]\!]^{(J)}$ = sharing interpolating $\left([\![x]\!]_i\right)_{i \in J}$
- Many different $[\![x]\!]^{(J)}$ $\Rightarrow$ many possible false positives

- "Degree-enforcing commitment scheme"
- Verifier $\rightarrow$ Prover : random $\{\gamma_j\}$
- Prover $\rightarrow$ Verifier : $[\![\xi]\!] = \Sigma_j \, \gamma_i \cdot [\![x_j]\!]$
- Before MPC computation

$$\Longrightarrow \quad \frac{1}{\binom{N}{\ell}} + p$$

TCitH vs. standard MPCitH

$$\ell = 1 \;\Rightarrow\; \text{Similar soundness: } \frac{1}{N} + p$$

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	MPCitH + seed trees + hypercube [AGHHJY23]	**TCitH** $\ell = 1$
Prover runtime	Party emulations: $\log N + 1$ Symmetric crypto: $O(N)$	Party emulations: 2 Symmetric crypto: $O(N)$

😀

fewer party emulations

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😀 😀

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much less symmetric crypto

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Size of tree	128-bit security: ~2KB 256-bit security: ~8KB	128-bit security: ~4KB 256-bit security: ~16KB	🙁

×2

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Getting rid of these limitations

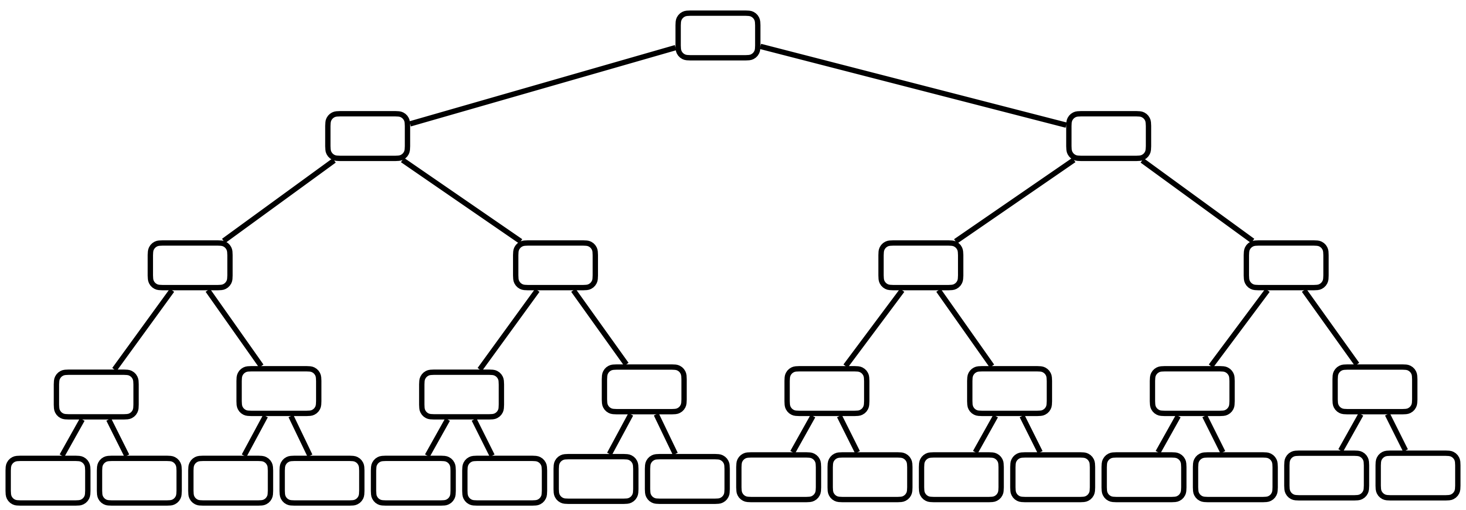
→ TCitH with GGM tree

TC-in-the-Head framework with GGM trees

TCitH with GGM trees

Step 1: Generate a replicated secret sharing [ISN89]

$x = r_1 + r_2 + \cdots + r_N$



$r_1 + r_2 + \cdots \quad \cdots + r_N = x + \Delta_x$

TCitH with GGM trees

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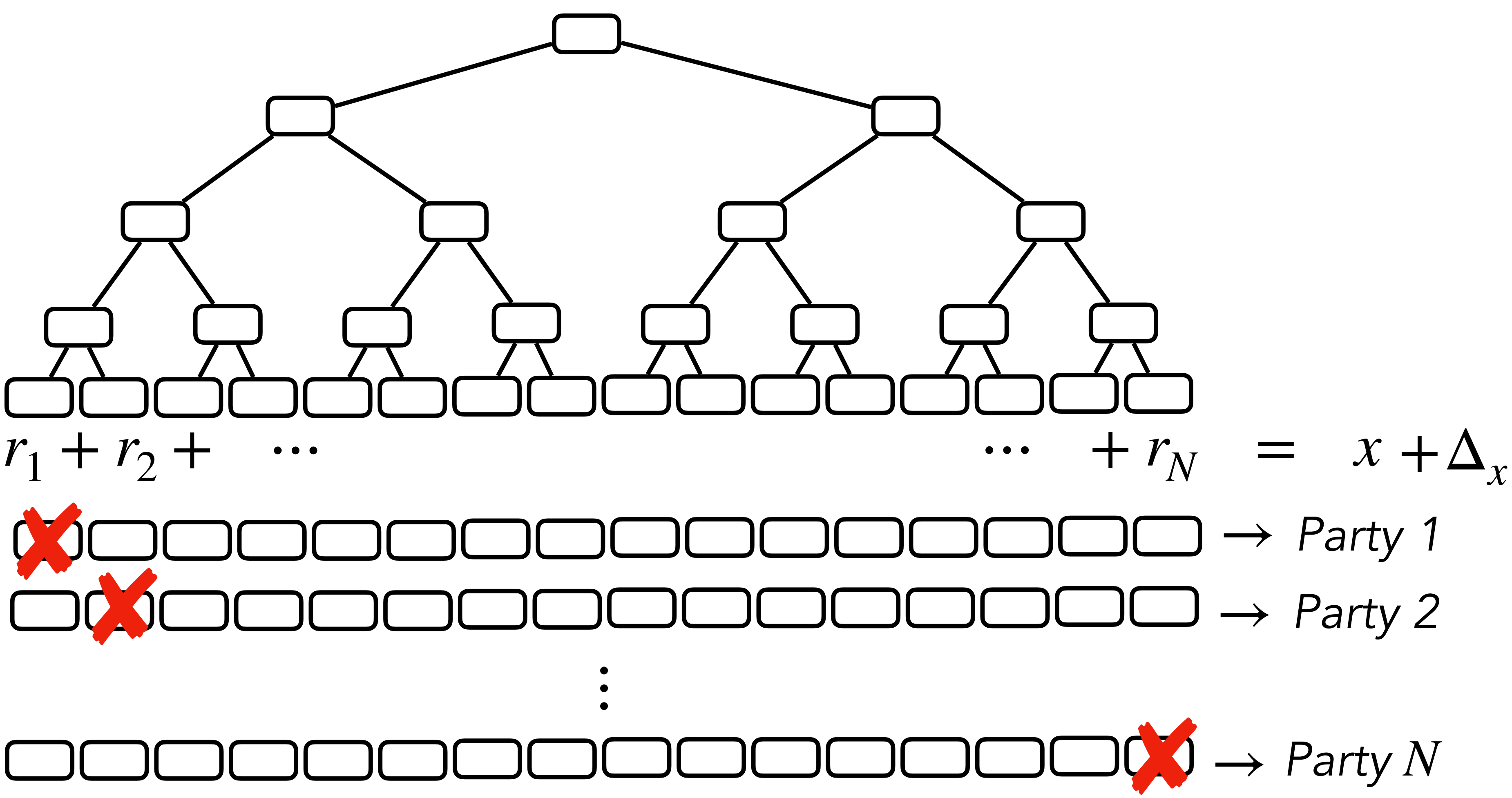

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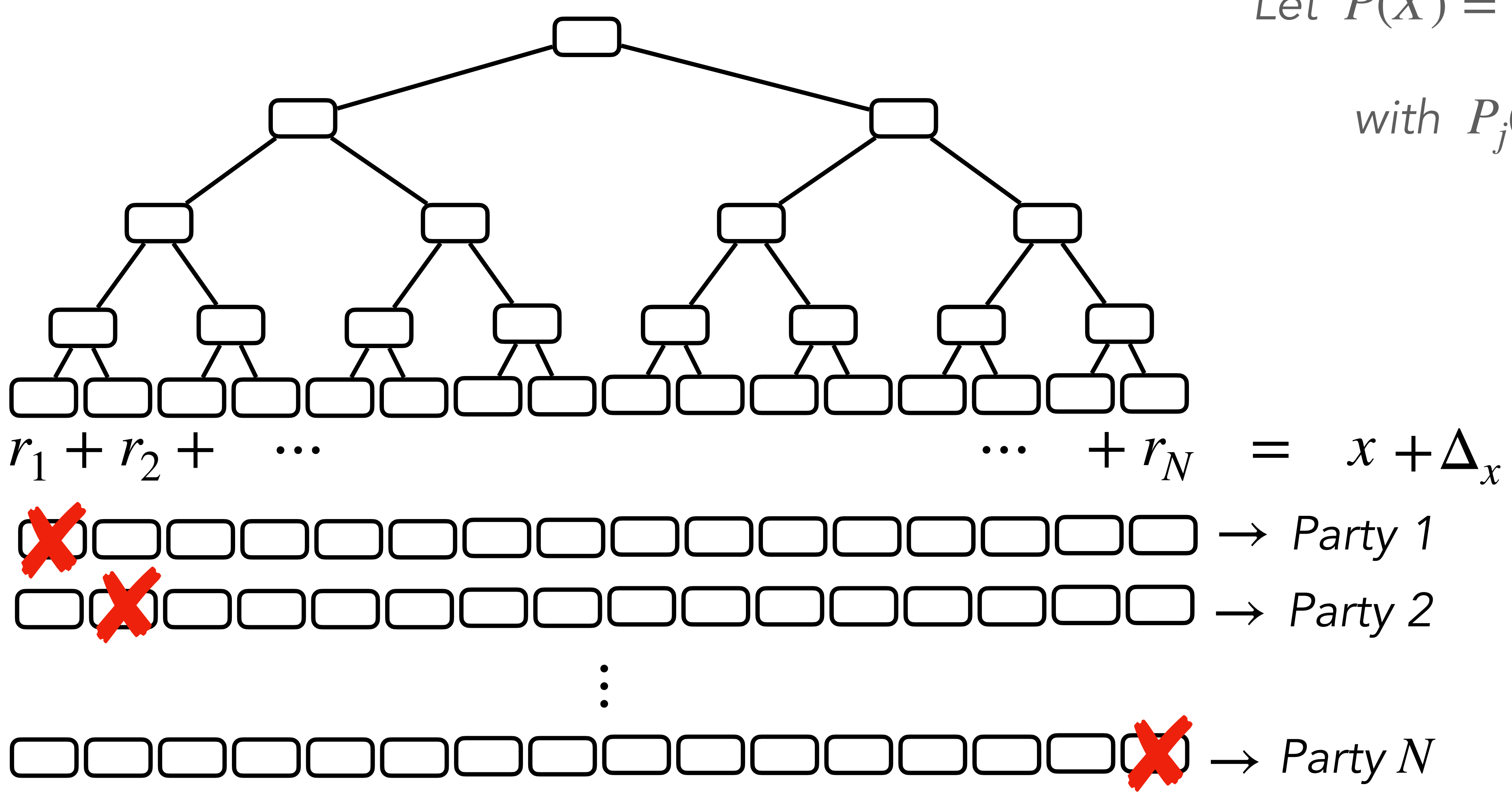

Step 2: Convert it into a Shamir's secret sharing [CDI05]

Let $P(X) = \Delta_x + \sum_j r_j P_j(X)$

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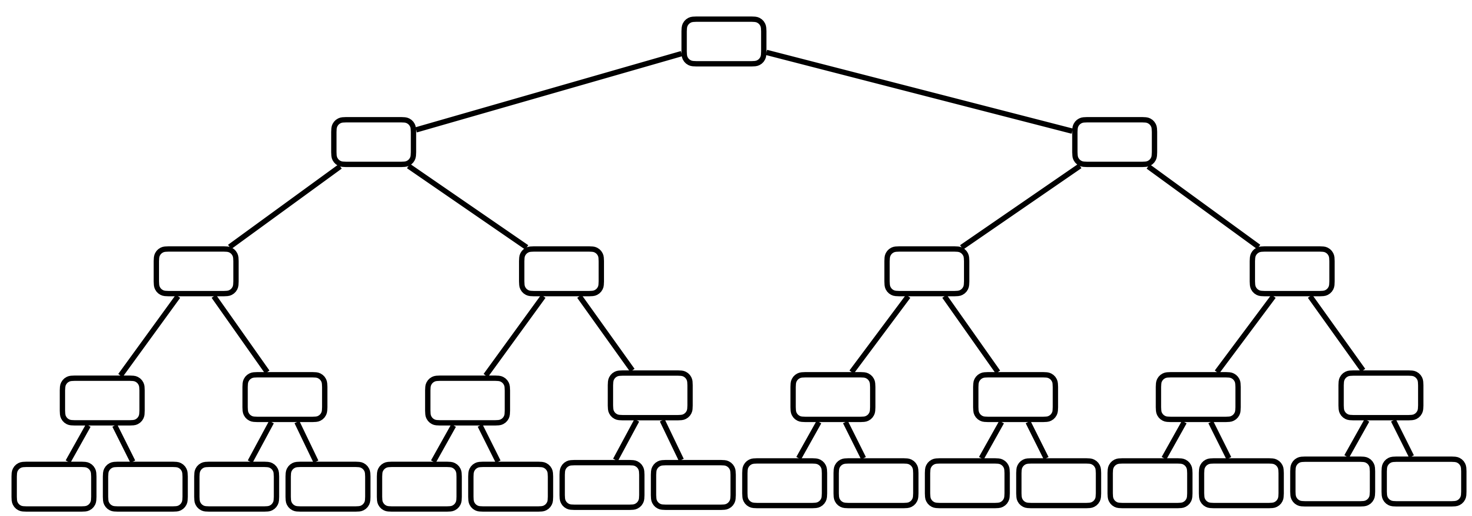

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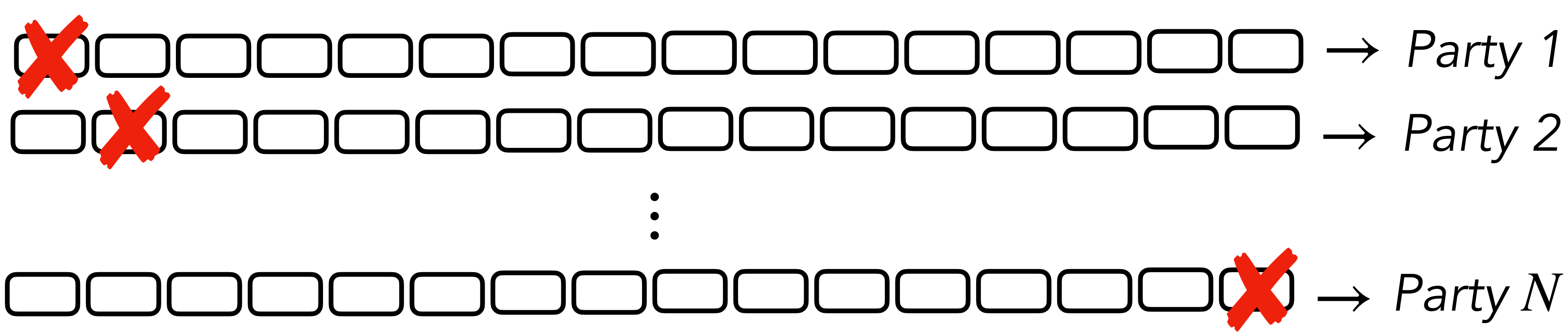

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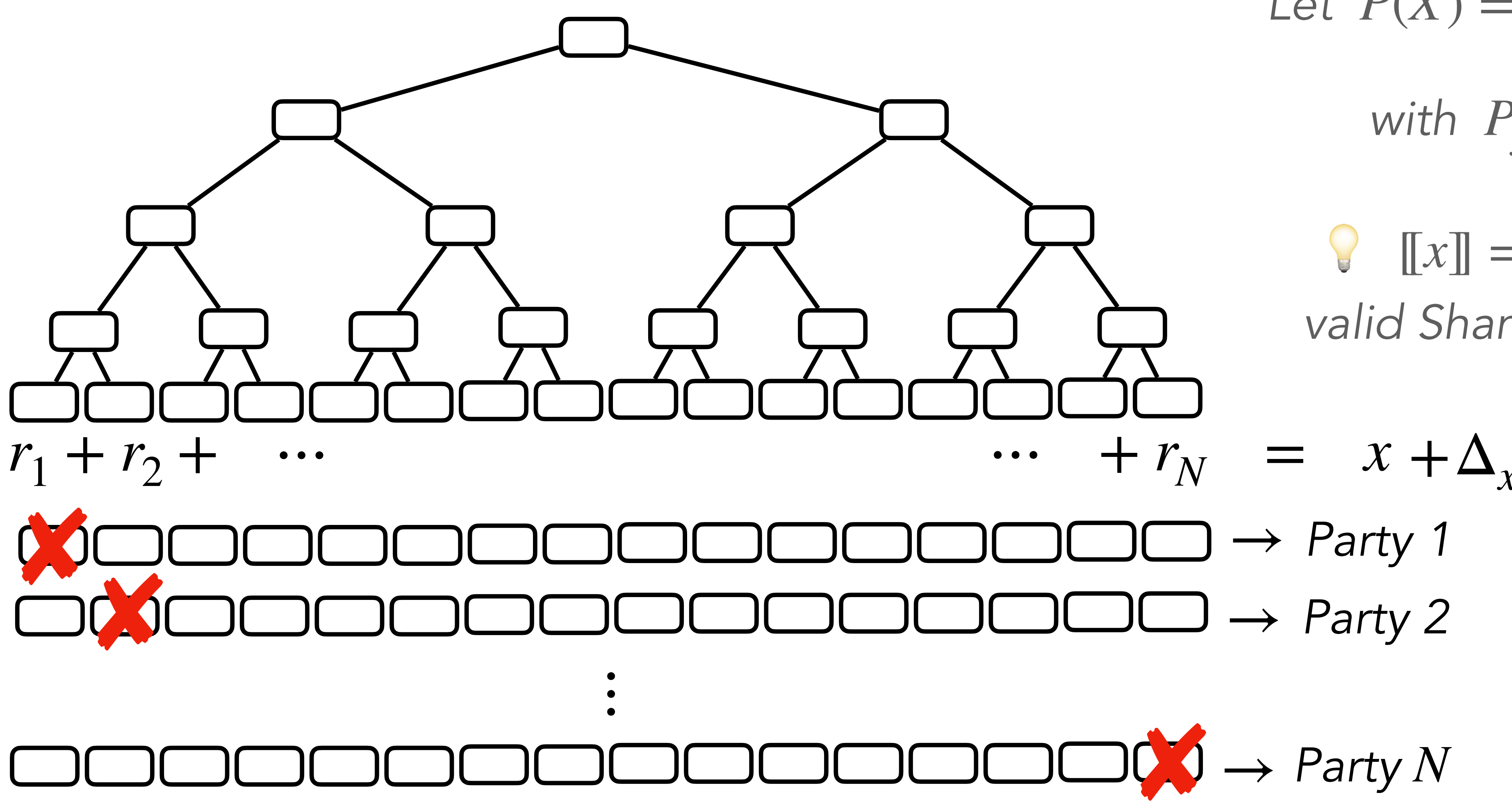

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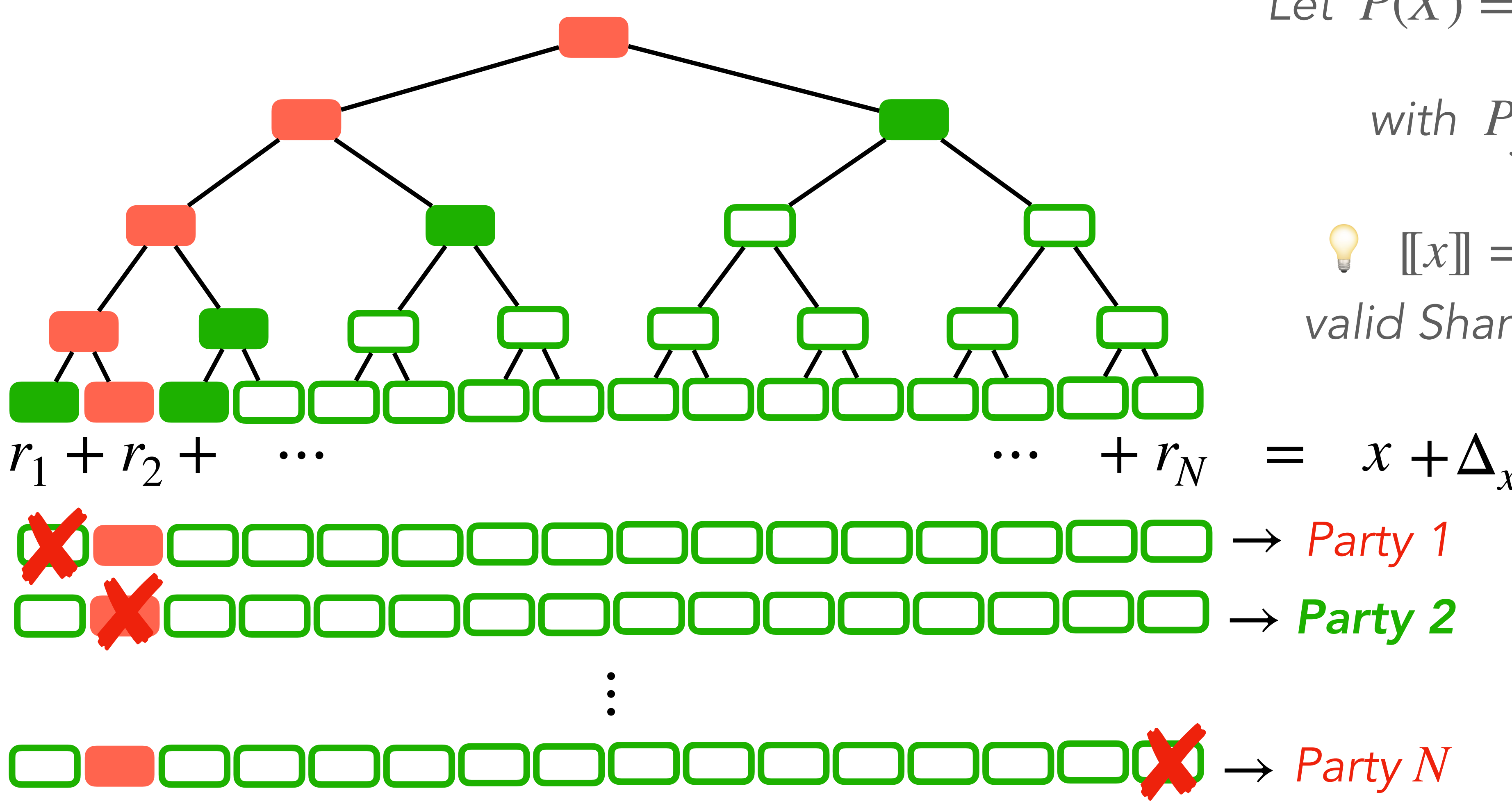

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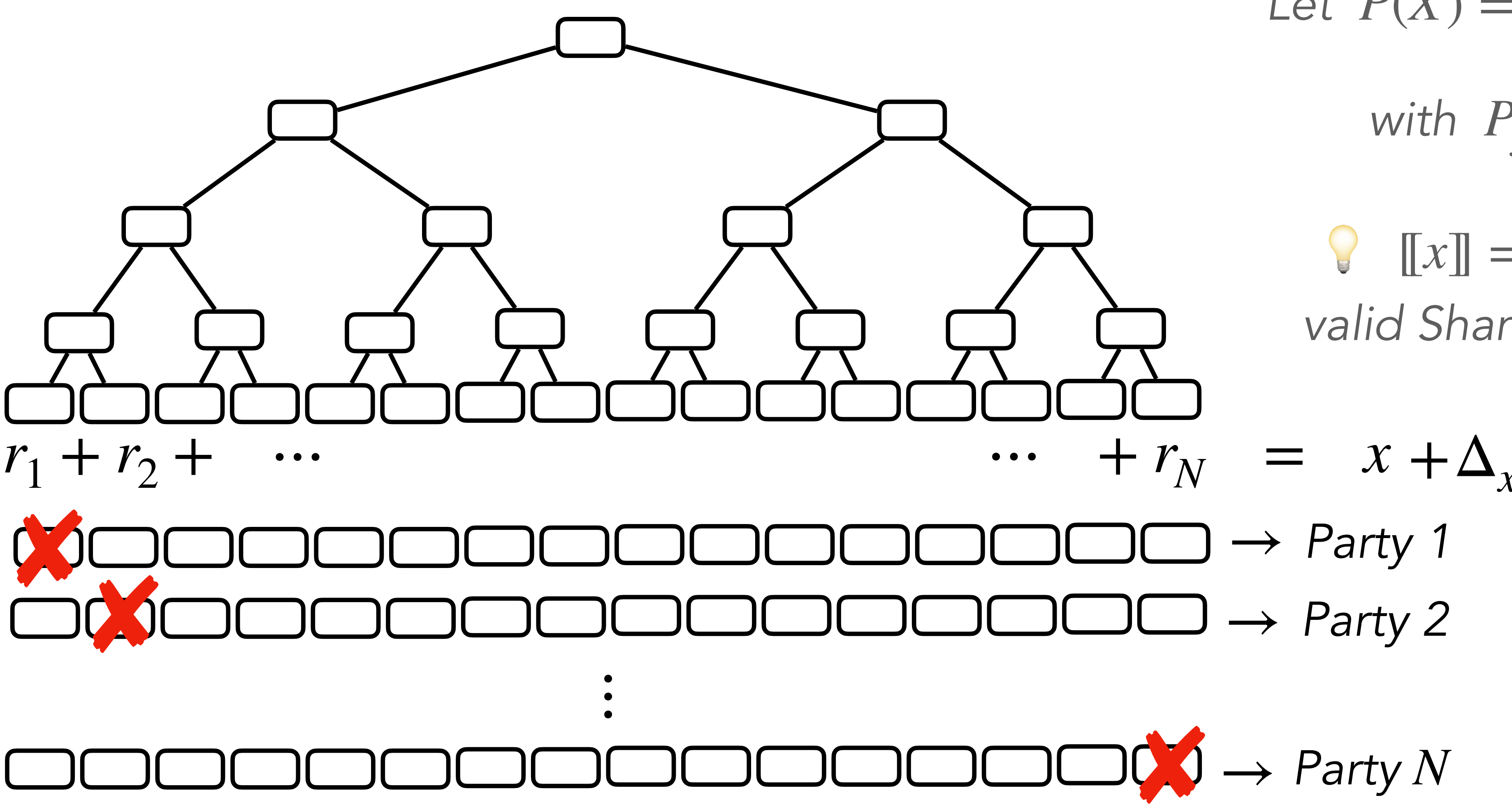

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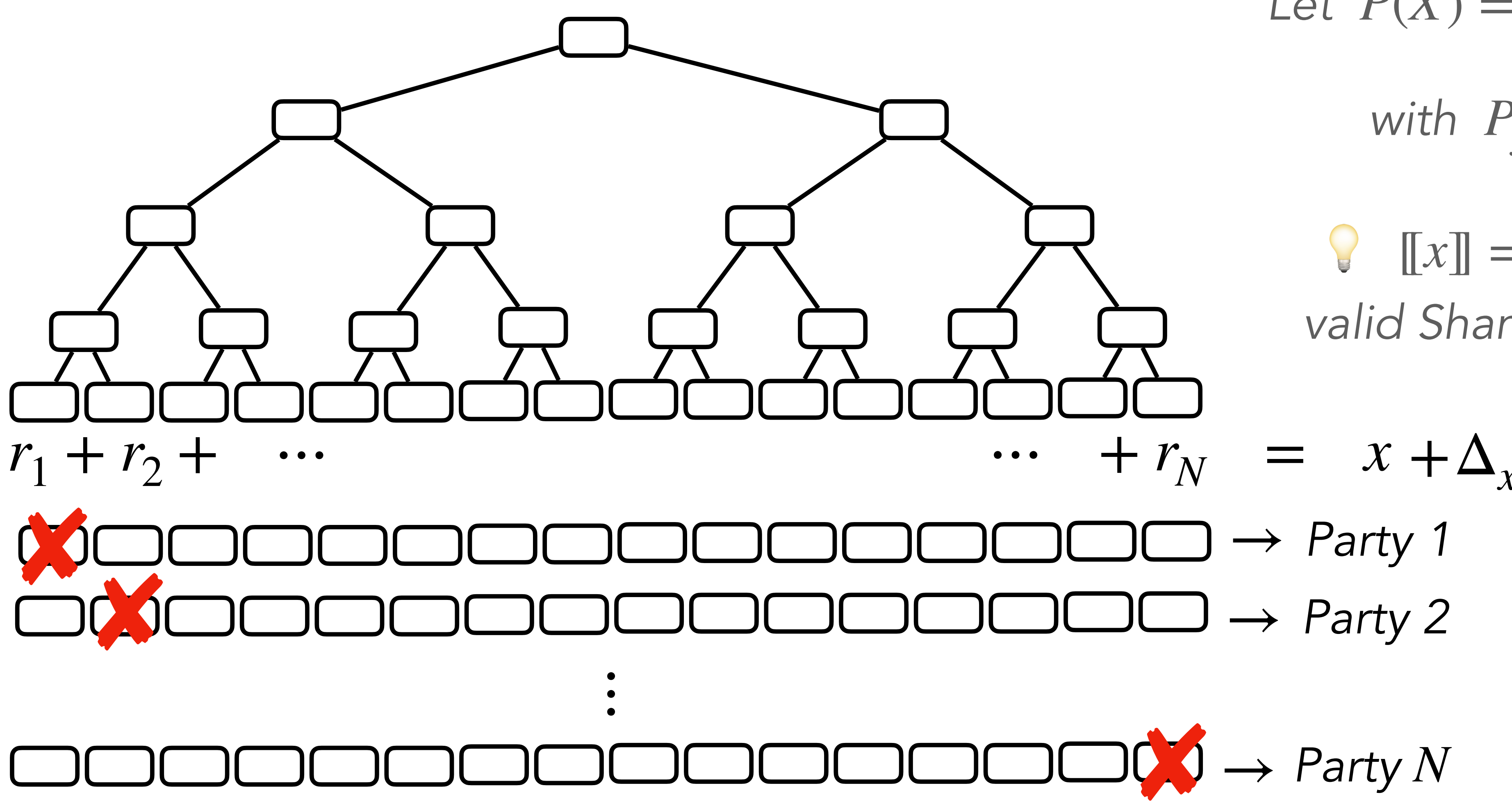

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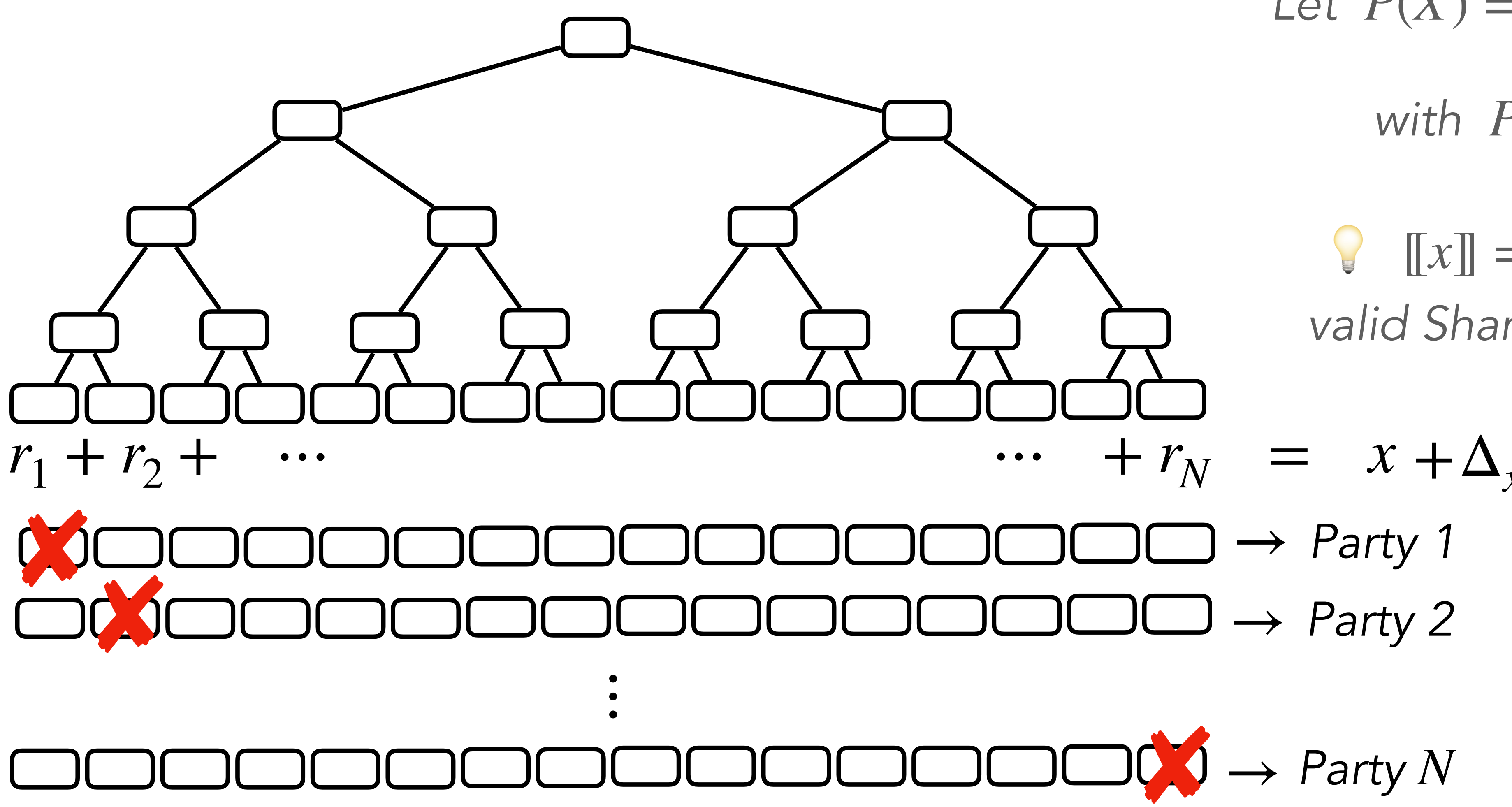

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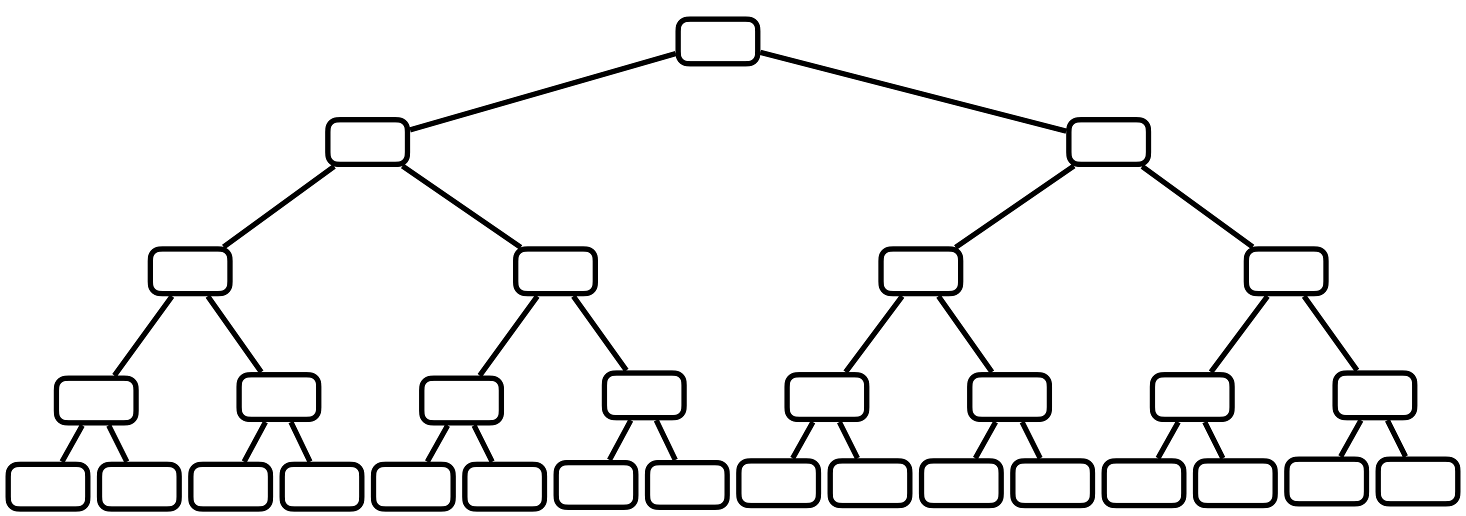

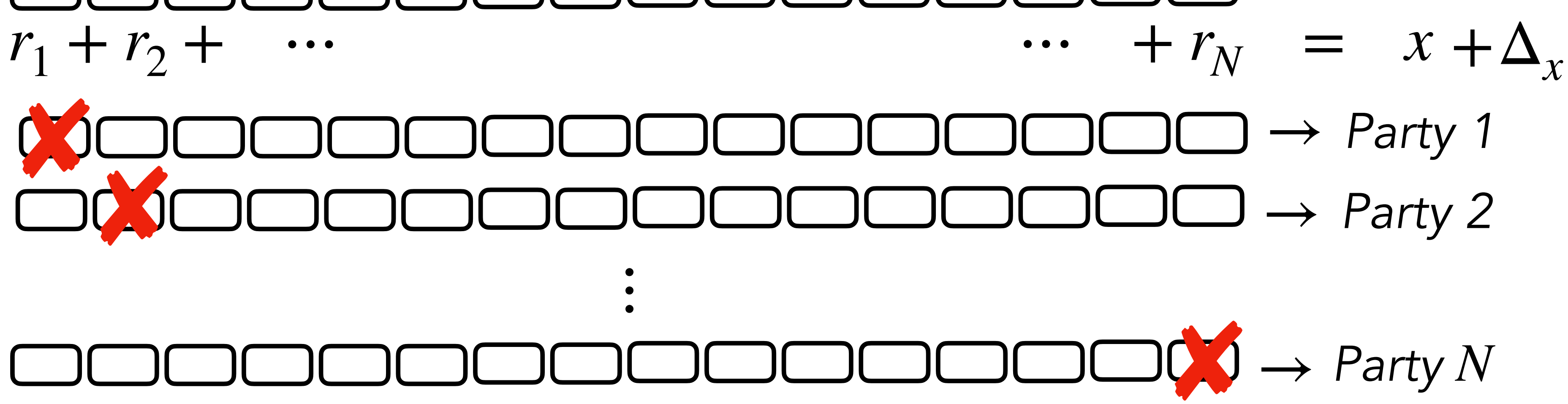

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🐌 Loose fast verification

Speedups for MPCitH candidates

	Additive MPCitH		TCitH (GGM tree)	
	Traditional (ms)	Hypercube (ms)	TCitH (ms)	Saving
Party emulations / repetition	N	$1 + \log_2 N$	2	

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AIMer	4.53	3.22	3.22	-0 %
Biscuit	17.71	4.65	4.24	-16 %
MIRA	384.26	20.11	9.89	-51 %
MiRitH-Ia	54.15	6.60	5.42	-18 %
MiRitH-Ib	89.50	8.66	6.66	-23 %
MQOM-31	96.41	11.27	8.74	-21 %
MQOM-251	44.11	7.56	5.97	-21 %
RYDE	12.41	4.65	4.65	-0 %
SDitH-256	78.37	7.23	5.31	-27 %
SDitH-251	19.15	7.53	6.44	-14 %

- Comparison based on a generic MPCitH library (libmpcith)
- Code for MPC protocols fetched from the submission packages

Using multiplication homomorphism & packed secret sharing

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$$[\![x]\!]^{(d)} \cdot [\![y]\!]^{(d)} = [\![x \cdot y]\!]^{(2d)}$$

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check $\alpha = 0$
false positive proba $1/|\mathbb{F}|$

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$$\frac{\binom{d_\alpha}{\ell}}{\binom{N}{\ell}} + p$$

Soundness error

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Soundness error

Here: $\ell \cdot \deg f_j$ $\left(\frac{1}{|\mathbb{F}|}\right)^{\#\alpha}$

Shorter signatures for MPCitH-based candidates

	Original Size	*Our Variant*	*Saving*
Biscuit	4 758 B	4 048 B	-15 %
MIRA	5 640 B	5 340 B	-5 %
MiRitH-Ia	5 665 B	4 694 B	-17 %
MiRitH-Ib	6 298 B	5 245 B	-17 %
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SDitH	8 241 B	7 335 B	-27 %
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* $N = 256$

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Biscuit	4 758 B	3 431 B	
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MiRitH-Ib	6 298 B	4 250 B	
MQOM-31	6 328 B	3 567 B	
MQOM-251	6 575 B	3 418 B	
RYDE	5 956 B	4 274 B	
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* $N = 256$ * $N = 2048$

Shorter signatures for MPCitH-based candidates

Two very recent works :

- Baum, Beullens, Mukherjee, Orsini, Ramacher, Rechberger, Roy, Scholl. *One Tree to Rule Them All: Optimizing GGM Trees and OWFs for Post-Quantum Signatures.* https://ia.cr/2024/490
  - General techniques to reduce the size of GGM trees
  - **Apply to TCitH-GGM** (gain of ~500 B at 128-bit security)

- Bidoux, Feneuil, Gaborit, Neveu, Rivain. *Dual Support Decomposition in the Head: Shorter Signatures from Rank SD and MinRank.* https://ia.cr/2024/541
  - New MPC protocols for TCitH / VOLEitH signatures based on **MinRank & Rank SD**

Using packed secret sharing

- Shamir's secret sharing can be packed
  - $P(\omega_1) = x_1 , \quad \dots , \quad P(\omega_s) = x_s$
  - $P(\omega_{s+1}) = r_1 , \quad \dots , P(\omega_{s+\ell}) = r_\ell$
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Here: $(\ell + s - 1) \cdot \deg f_j$

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- $[\![x]\!] + [\![y]\!] =$ sharing of $(x_1, \dots, x_s) + (y_1, \dots, y_s)$
- $[\![x]\!] \cdot [\![y]\!] =$ sharing of $(x_1, \dots, x_s) \circ (y_1, \dots, y_s)$
- Packed sharing & Merkle trees $\approx \div$ witness size by s

  $\Rightarrow$ interesting for statements with "medium size" witness

Here: $(\ell + s - 1) \cdot \deg f_j$

$$\frac{\binom{d_\alpha}{\ell}}{\binom{N}{\ell}} + p$$

Soundness error

Using packed secret sharing

- Shamir's secret sharing can be packed
  - $P(\omega_1) = x_1 , \quad \ldots , \quad P(\omega_s) = x_s$
  - $P(\omega_{s+1}) = r_1 , \quad \ldots , P(\omega_{s+\ell}) = r_\ell$
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- Packed sharing & Merkle trees $\approx \div$ witness size by s

  $\Rightarrow$ interesting for statements with "medium size" witness
- E.g. an ISIS statement $\vec{t} = A \cdot \vec{e}$ with $\| \vec{e} \|_\infty \leq \beta$

Here: $(\ell + s - 1) \cdot \deg f_j$

$$\frac{\binom{d_\alpha}{\ell}}{\binom{N}{\ell}} + p$$

Soundness error

TCitH-GGM vs. TCitH-MT

TCitH-GGM	*TCitH-MT*
🎄 Smaller tree	🌲 Larger tree (~x2)

TCitH-GGM vs. TCitH-MT

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☑️ Naturally enforce degree of committed sharings	🔁 Need degree enforcing commitment (+1 round)

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☑️ Naturally enforce degree of committed sharings	🔁 Need degree enforcing commitment (+1 round)
🎯 Better for "small-size" statements	🎯 Better for "medium-size" statements

Application: post-quantum ring signatures

Post-quantum ring signatures

- Secret key w
- One-way function f
- Public key $y = f(w)$
- MPC protocol $\Pi : [\![w]\!] \mapsto 0/1$

TCitH → *FS*

✍️ signature scheme

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TCitH

FS

✍️ signature scheme

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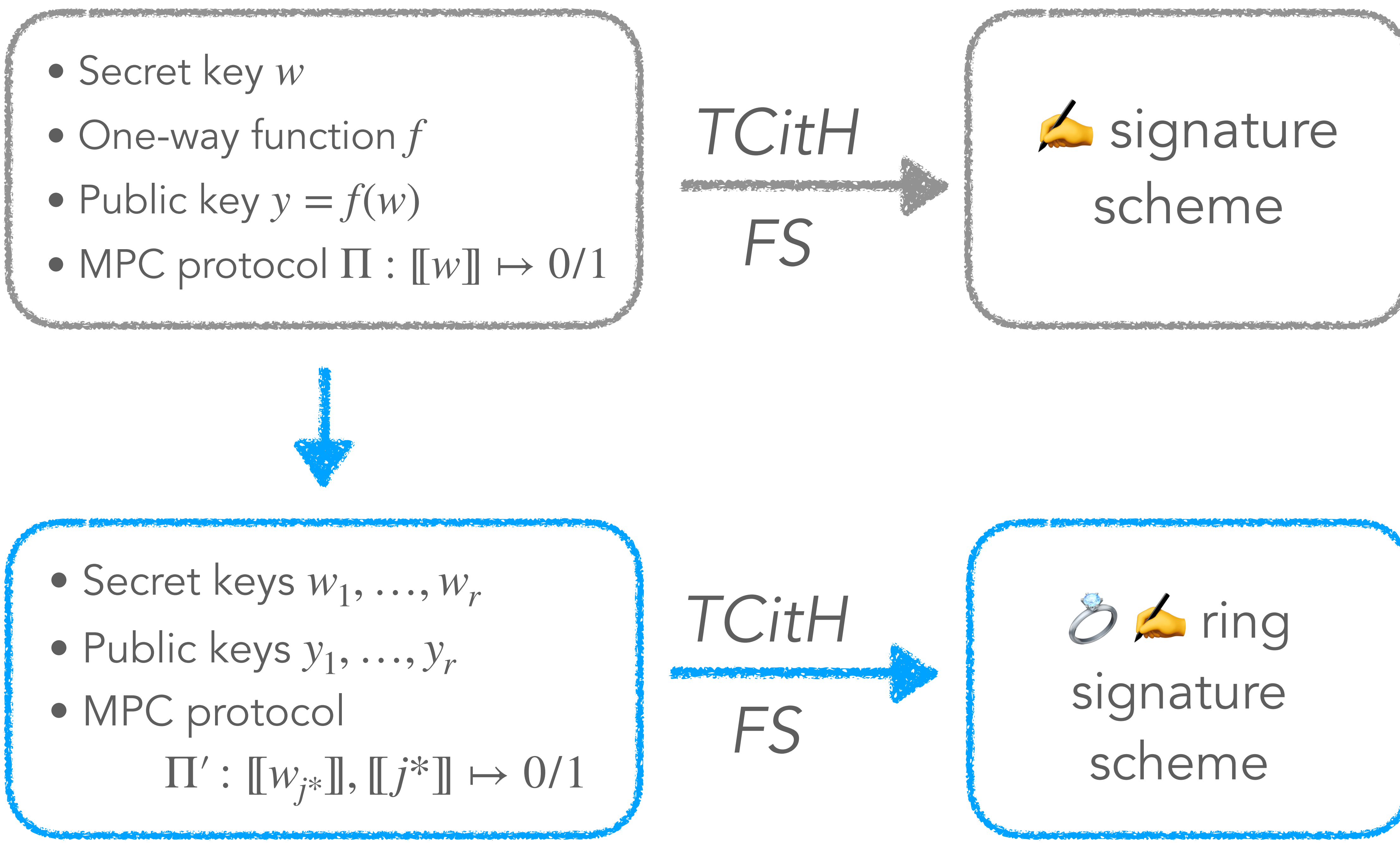
$$\Pi' : [\![w_{j^*}]\!], [\![j^*]\!] \mapsto 0/1$$

Post-quantum ring signatures



Post-quantum ring signatures

💡 Idea:

- One-hot encoding of $j*$

$$s = (0,\ldots,0, s_{j*} := 1, 0,\ldots,0)$$

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Post-quantum ring signatures

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Post-quantum ring signatures

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🤔 Problem: including $[\![s]\!]$ to the witness $\Rightarrow \mathcal{O}(r)$ signature size

🛠 Solution: $[\![s^{(1)}]\!], \ldots, [\![s^{(d)}]\!]$ s.t. $s = s^{(1)} \otimes \cdots \otimes s^{(d)}$

$\Rightarrow \mathcal{O}\left(d\sqrt[d]{r}\,\right)$ signature size $\Rightarrow \mathcal{O}(\log r)$

Post-quantum ring signatures

Protocol Π'

Input: $[\![w]\!], [\![s^{(1)}]\!], \ldots, [\![s^{(d)}]\!]$

Post-quantum ring signatures

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TCitH / FS

💍 ✍️ ring signature scheme

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⚠️ Sharing degrees increase

Post-quantum ring signatures

#users		2^3	2^6	2^8	2^{10}	2^{12}	2^{20}	Assumption	Security
Our scheme	2023	4.41	4.60	4.90	5.48	5.82	8.19	MQ over $\mathbb{F}_{251}$	NIST I
Our scheme	2023	4.30	4.33	4.37	4.45	4.60	5.62	MQ over $\mathbb{F}_{256}$	NIST I
Our scheme	2023	7.51	8.40	8.72	9.36	10.30	12.81	SD over $\mathbb{F}_{251}$	NIST I
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Our scheme	2023	6.81	6.84	6.88	6.96	7.12	8.27	AES128-EM	NIST I
KKW [KKW18]	2018	-	250	-	-	456	-	LowMC	NIST V
GGHK [GGHAK22]	2021	-	-	-	56	-	-	LowMC	NIST V
Raptor [LAZ19]	2019	10	81	333	1290	5161	-	MSIS / MLWE	100 bit
EZSLL [EZS+19]	2019	19	31	-	-	148	-	MSIS / MLWE	NIST II
Falafl [BKP20]	2020	30	32	-	-	35	-	MSIS / MLWE	NIST I
Calamari [BKP20]	2020	5	8	-	-	14	-	CSIDH	128 bit
LESS [BBN+22]	2022	11	14	-	-	20	-	Code Equiv.	128 bit
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Post-quantum ring signatures

Application to MQ, SD, AES

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Size range: 5–13 kB for |ring|=2^{20}

Post-quantum ring signatures

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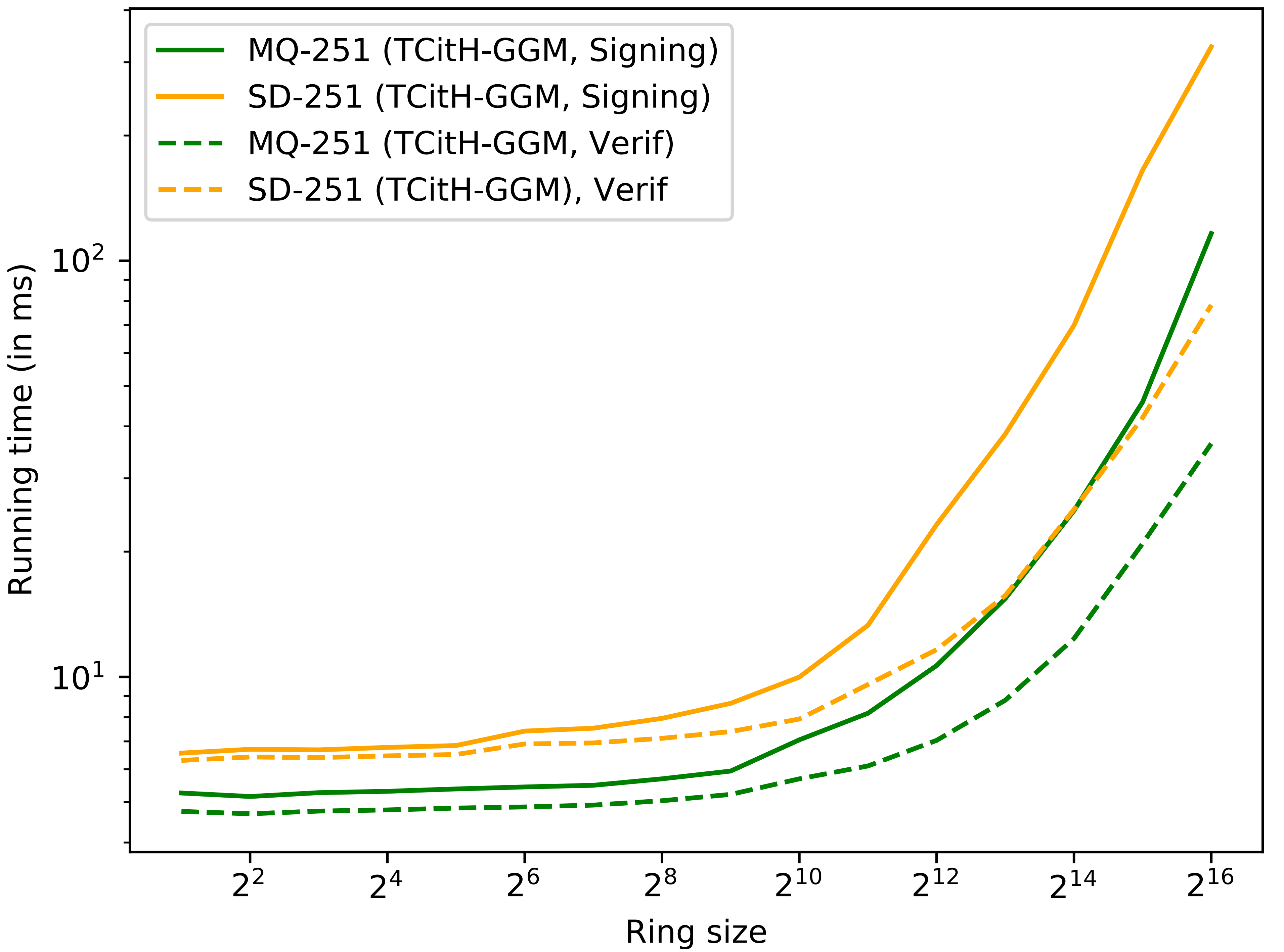
Previous works:

$\geq$ 14 kB for |ring|=2^{10}

no / slow implementations

Post-quantum ring signatures

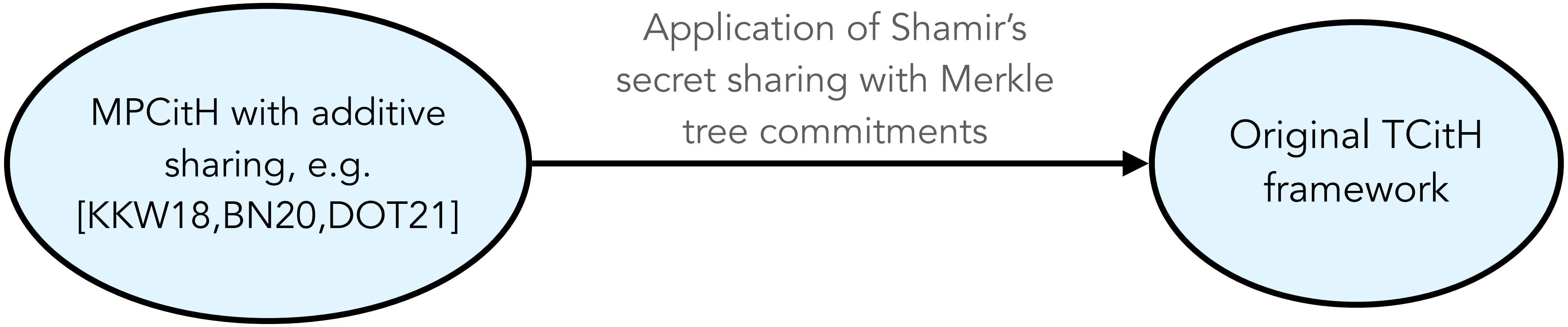


Relation to other proof systems

Connections to other proof systems

MPCitH with additive sharing, e.g. [KKW18,BN20,DOT21]

Connections to other proof systems



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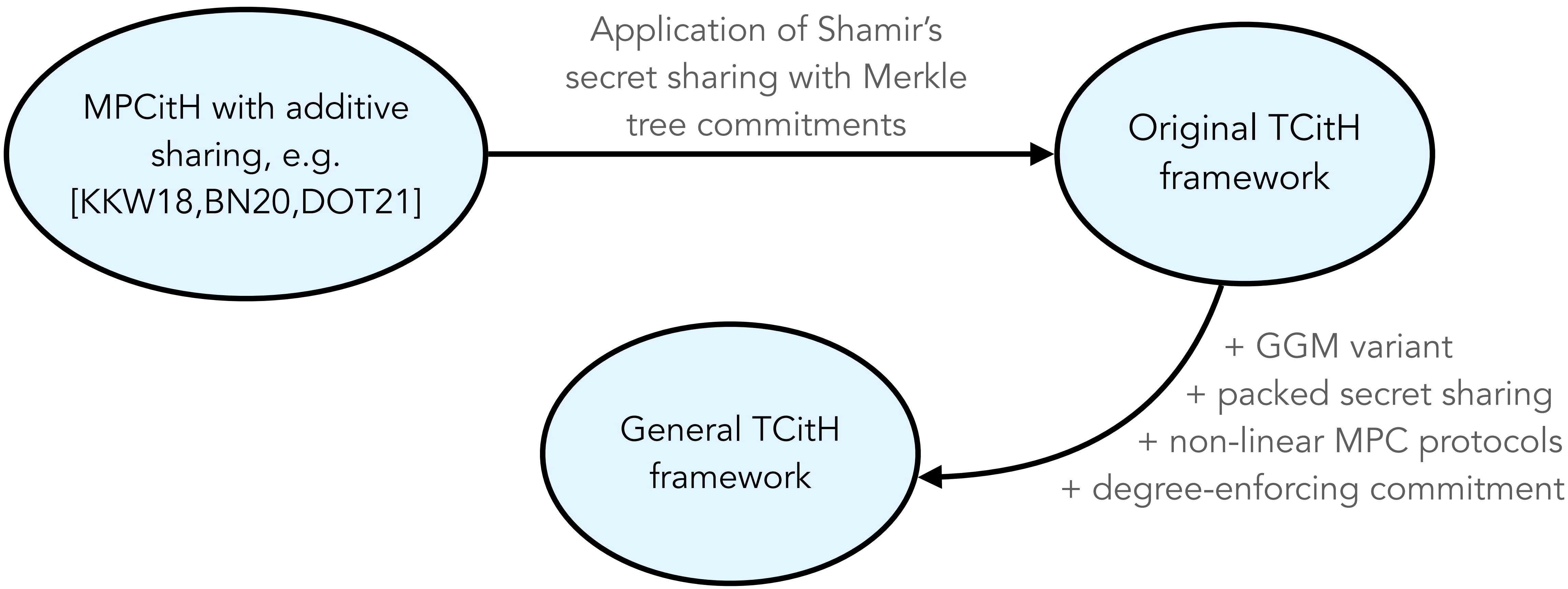

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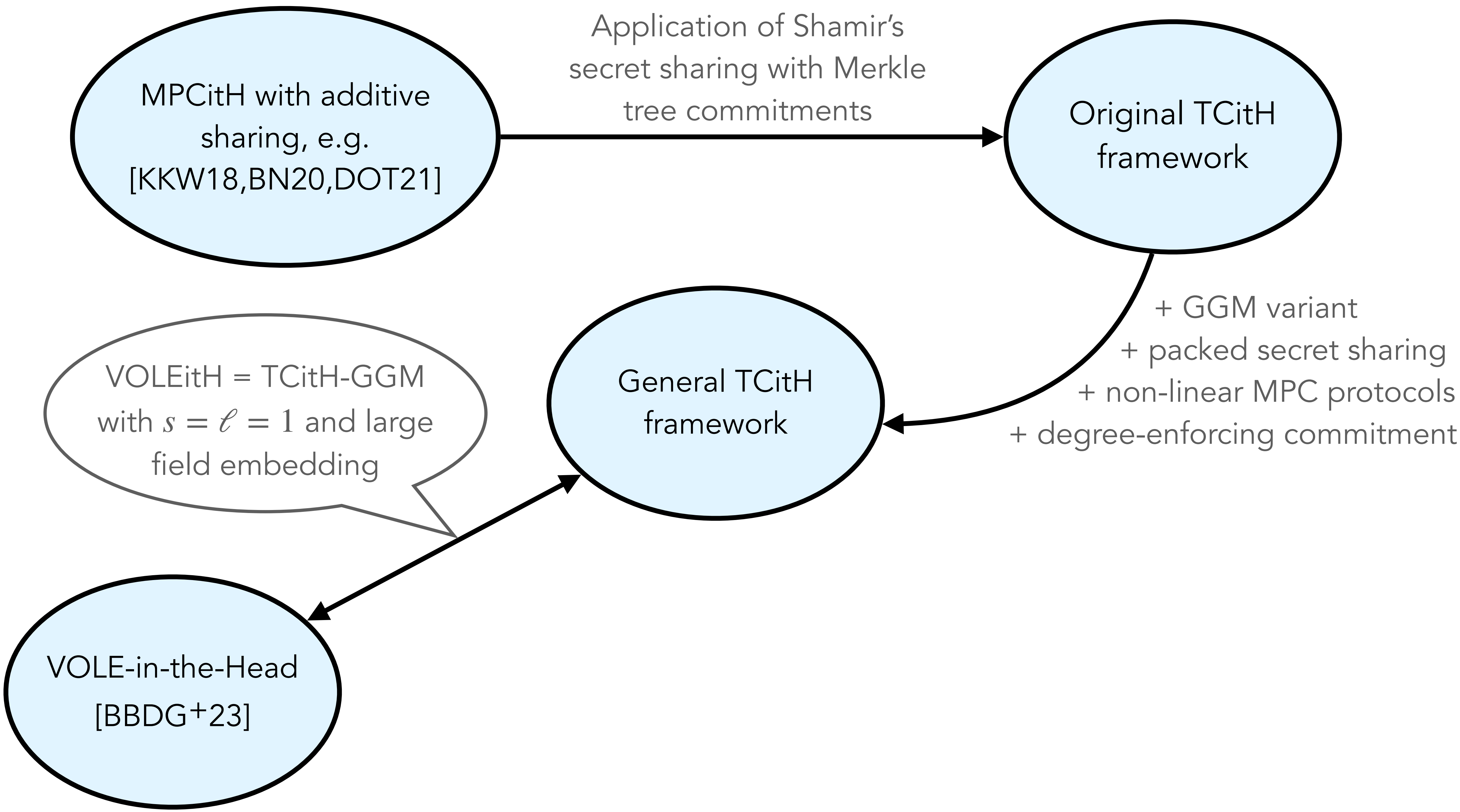

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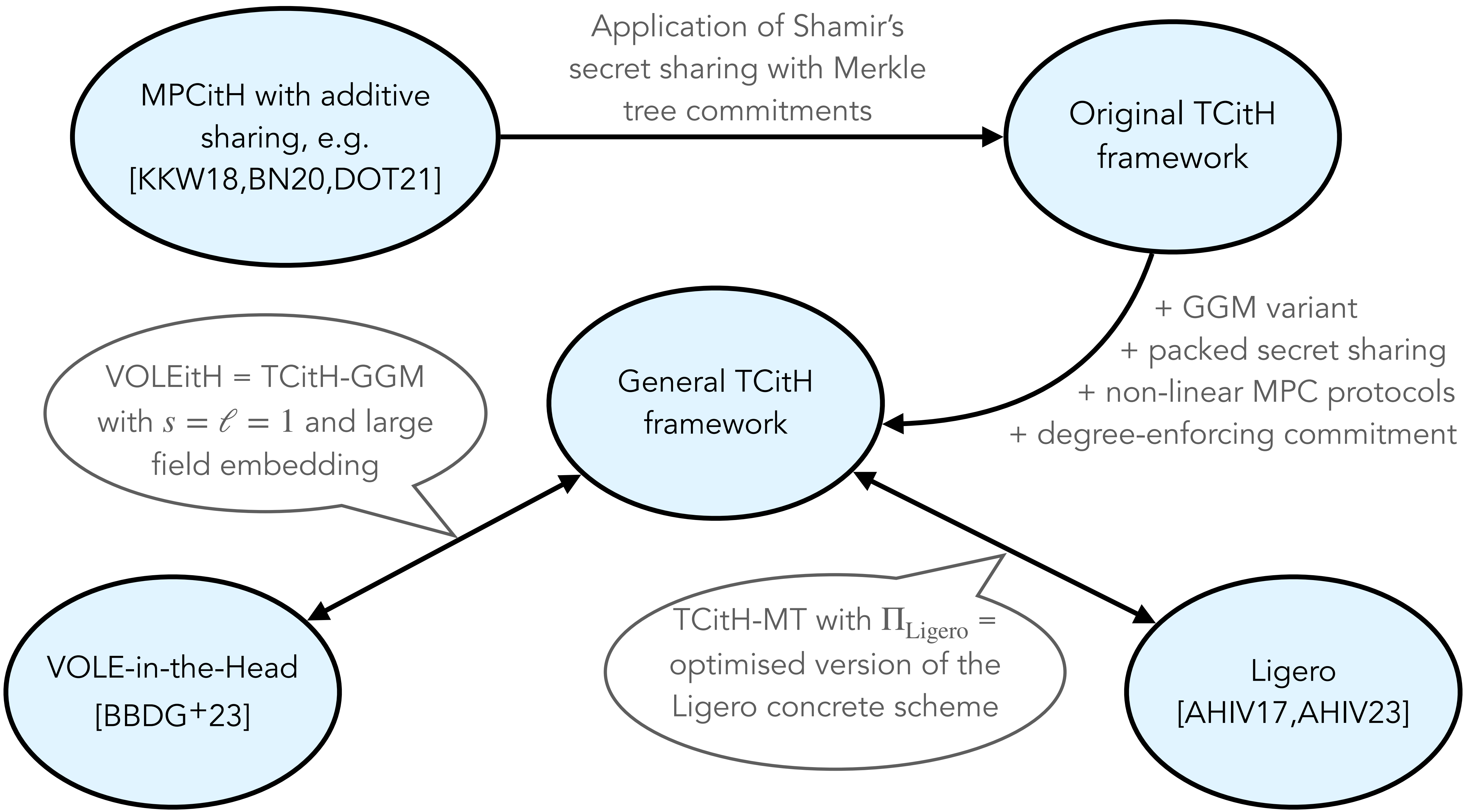

Thank you!

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